

Prediction of Risk of Airborne Transmitted Diseases based on the Wells-Riley Equation: Applying in the Metro Hall Environments

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Abstract: The recent years have witnessed the increasing attention aroused by airborne transmitted diseases. Increasing evidence has shown that indoor air quality has a significant impact on people's health, as well as the transmission of infectious diseases. As a result, more and more experts pay attention to the topic of airborne transmitted diseases. To improve the understanding of the risk of airborne transmitted diseases, in this paper we have reviewed the previous risk prediction model and tried to develop an improved model. The model, which has taken the size of space, the ventilation of the space, and the time of co-location, are also applied in a realistic model based on the system dynamic method, simulating the condition of passengers' going into the station hall. According to the model and simulation outcoming, expanding the volume of room and ventilation can reduce the risk of infection. However, the co-location has a positive correlation with the infectious risk.

Keywords: Airbone Transmitted Diseases; Wells-riley Model; System Dynamics; Risk

1. Introduction

In recent years, with the increasing evidence that indoor air quality has a significant impact on the health of individuals and plays an important role in the transmission of infectious diseases, the airborne transmission of infectious diseases has become a topic of concern^[1]. According to available information, the virus can be transmitted by droplets, aerosol, direct contact, indirect contact, and other ways, but airborne transmission is still the main factor^[2]. The modern urban space environment is characterized by high density and high mobility. The rapid spread of viruses in this environment poses a great challenge to

people's healthy life. Since the SARS outbreak in 2003, the importance of indoor air safety has been recognized more and more. Although SARS has passed, airborne diseases caused by anthrax or smallpox, as well as possible bird flu outbreaks, still threaten humanity. What's more, according to the World Health Organization, tuberculosis, an airborne infection, kills 1.7million people each year^[3].COVID- 19 (Corona Virus Disease 2019), a classic example of an airborne disease, is a particular concern at the moment, once again making airborne diseases a worldwide health problem^[4].

2. Literature Review

Many studies have shown that air infectious diseases are related to ventilation, air distribution, and other factors. Understanding the transmission route of air infectious diseases and accurately predicting the infection probability and influencing factors of air infectious diseases are helpful to take measures to reduce the infection probability.

At present, China has studied the use of microsimulation modeling methods to analyze the passenger flow in subway stations. Hu Mingwei^[5,6] et al. Used the microsimulation model to evaluate subway stations' passenger flow organization and management. Ji^[7,8] et al. Proposed an improved social force model and cellular automata model by observing and analyzing the passenger flow in subway stations. Trivedi^[9] and others analyze and calculate the total time of subway station passengers from waiting area to boarding based on the agent modelingmethod. Compared with the microsimulation model, the system dynamics method can carry out the global analysis of the system and has the advantages of fast modeling speed and less workload for the comparison and selection of different schemes. Chen Chunan^[10] et al. Used

the system dynamics model to deduce and simulate the passenger flow of subway stations. The results show that the model method can predict and analyze passenger flow. By studying the statistical prediction model of infectious diseases, Zhang Yi^[11] and others put forward the concept of travel susceptibility and calculated the susceptibility of different means of transportation respectively.

China's novel coronavirus pneumonia (COVID-19) joint study report is based on the new crown virus epidemic^[12]. The incubation period of new crown pneumonia is generally 1~14 D, with an average incubation period of 5~6 D, and there may be asymptomatic carriers taking the subway, so, therefore, the 1~14 is the most common risk factor for new pneumonia. During the epidemic prevention and control period, the subway operator should also consider the impact of measures on the infection probability of passengers when formulating control strategies. When New Coronavirus spreads through droplets and air, it is affected by many factors, such as the propagation environment, quantity and type of pathogen, and evaporation and dispersion characteristics of droplets in different indoor environments. Because of the inaccurate calculation of some of the factors and the late start of some factors, It is difficult to accurately predict the infection probability of respiratory infectious diseases from the mechanism. At present, the prediction models of respiratory infectious diseases are mostly based on statistics to predict the infection risk and transmission probability of diseases, such as Si, SIS, sir, SIRS, and SEIR models^[13,14]. Existing studies have shown that most of the relevant control strategies of the subway are based on passenger safety, traffic efficiency, and the carrying capacity of the subway. The impact indicators to be considered in the strategy selection under emergencies need to be improved.

Although this problem exists all over the world, there are relatively few studies on this problem to quantify the risk of air transmission in closed space, and most previous studies come from the work of wells and Riley et al, using an analytical method expression called Wells Riley equation^[15]. Although this has been applied to many areas of risk analysis studies, including the assessment of personal protective equipment, the risk of tuberculosis in buildings,

and the spread of tuberculosis, there are many limitations from the coated bacillus. In most cases, the analysis assumes that the air is fully mixed with the indoor air, resulting in the uniform concentration of biological aerosol in the whole space, so the proximity effect between infected and susceptible persons is not considered. This model can also ignore the inherent random effects in small groups, resulting in only quantifying the average risk rather than the expected range.

This paper aims to develop a model to evaluate the probability of airborne and droplet-borne diseases in confined spaces. Factors to be considered include the size of the space, the ventilation of the space, the time of coexistence, as well as the distribution of personnel, and airflow conditions.

3. Models

3.1 Basic Model

3.1.1 The mass action model

The MA (mass action) model is expressed as

$$C=rIS \quad (1)$$

In the equation, C represents the number of people who will have been newly infected; r represents the effective connection rate; I represents the number of people who have been infected; S represents the number of susceptible people in the space. The equation (1) can be used in the prediction of the infectious rate during an outbreak.

Based on the previous equation, if we take it as an assumption that the droplet nuclei are distributed evenly in the whole space, it can be deduced the number of quanta one person breathes in the air, and thus calculate the effective contact rate, which can be expressed as

$$r = \frac{qpt}{Q} \quad (2)$$

In the equation q represents the quanta production rate of one infection; p is the breath ventilation of susceptible individuals (m^3/h); Q is the constant rate of ventilation in an indoor environment (m^3/h); t represents the time when the susceptible individual is exposed in the environment (h).

Substitute equation (1) into equation (2), it can be inferred that the infection rate is

$$P = \frac{C}{S} = rI = \frac{qpt}{Q}I \quad (3)$$

However, the equation does not take the average infectious rate into account, it means while quanta are high, the outcome of infectious rate might be higher than 1, which is unreliable. Based on the Wells theory, Riley developed the Wells-Riley equation in 1978, which can be expressed as

$$P = \frac{C}{S} = 1 - \exp\left(-\frac{Ipqt}{Q}\right) \quad (4)$$

which had successfully predicted an outbreak of one disease.

3.1.2 Assumptions

The Wells-Riley model is based on the following assumptions:

(1) The droplet nuclei are distributed evenly in the air of the whole room, which means the air is completely mixed with equal concentrations throughout the room, leading to the same infectious rate at any place in the room.

(2) The concentration of droplet nuclei is stable during the whole infection time, in another word, the amount of pathogen exhaled by one infected person, the number of an infected person, and the ventilation volume are stable during the whole infection period.

(3) Do not take the mortality rate of biological viruses before they are ventilated out of the room into account.

(4) Do not take the number of droplet nuclei removed from the room air by leakage, filtration, or sedimentation into account.

3.1.3 The Development of Model

The development of the Wells-Riley model is improved around the assumptions of the Wells-Riley equation, to make the predicted model more fitted with the actual situation.

Seppanen et.al (2006) took the air filter and particle settling effect into account and obtained the following equation

$$P = \frac{C}{S} = 1 - \exp\left(-\frac{Ipqt/v}{Vn_v + n_f + n_d}\right) \quad (5)$$

In the equation, V is the volume of the room; n_v is the number of air changes; n_f represents the product of return air volume and filter efficiency; n_d is the number of droplet nuclei settling on indoor surfaces.

Considering the effect of facemask, Fennelly et.al have developed the Wells-Riley model

into

$$P = \frac{C}{S} = 1 - \exp\left(-\frac{Ipqt\theta}{Q}\right) \quad (6)$$

According to the equation, θ refers to the permeability coefficient of the facemask, whose value range is 0 to 1. While θ equals 1, it means there is no facemask or the prevention effect of facemask is none.

Rudnick et.al have also developed the Wells-Riley model, to predict the infectious probability under unstable situations, using the concentration of CO₂ as the index of exhaled gas exposure. While there is no other origination of CO₂, the concentration of CO₂ can reflect the breathing of an individual and the ventilation situation. The model, which is suitable for the prediction of unstable and bad-ventilation environments, can be expressed as

$$P = \frac{C_{in}}{S} = 1 - \exp\left(-\frac{\bar{f}pqt}{S+1}\right) \quad (7)$$

The \bar{f} is defined as the percentage of a person's exhaled air that is reabsorbed.

$$\bar{f} = \frac{V_e}{V} = \frac{C_{in} - C_o}{C_a} \quad (8)$$

In the equation, C_a represents the volume percentage of CO₂ in exhaled gas; V_e is the equivalent volume of exhaled air in one room; C_{in} and C_o are the volume fractions of indoor and outdoor CO₂.

3.2 Improved Model

According to the previous research, the researchers mainly focus on a constant period of infection, ignoring the environment and ventilation condition. On the other hand, the time of co-location, that is, the time when the susceptible person stays aside from an infected person may also be an important factor of infection. In this research, we consider the size of the space, referring to the volume of a room, the ventilation level, and the time of co-location as parameters and build an improved model.

3.2.1 Extra Symbols

Based on the improved model, here are some extra symbols.

Considering the size of the space, we have set the V as the volume of the room. To numerically represent the level the ventilation

of one room, we defined the n_v as a symbol.

$$n_v = \frac{Q}{V} \tag{9}$$

Q refers to the ventilation amount of one room during a specific period, and V represents the volume of the room.

Taking the time of co-location into account, to distinguish the time from the original time of exposure in the equation, we set μ as the representative of co-location time, which means the percentage of time when a potential target of infection stays nearby an infected one of the whole period. As a result, the value range of α is 0 to 1.

3.2.2 Additional Assumptions

Based on the original model, the improved model has the following additional assumptions.

(1) The indoor space has an instant ventilation system, which periodically exchanges part of the air in the room.

(2) The performance changes linearly with ventilation rate, volume, and co-location time.

3.2.3 The Improved Model

The improved model, which has taken the size of the space, the ventilation level, and the co-location time into account, can be expressed as

$$\frac{C}{S} = 1 - \exp\left(-\frac{\alpha I p q t / V}{n_v}\right) \tag{10}$$

In the equation, the time of exposure in an infectious environment is replaced by the co-location time, which reveals the impact of staying nearby an infected person rather than simply locating in a room with the airborne transmitted disease.

3.2.4 Discussions

The quanta produced by different diseases and infected people are different. When an infected person exhales an enormous number of pathogens that reaches a high quanta value, the infected person, who is called a super spitter, has a higher probability to infect others and causing disease outbreaks

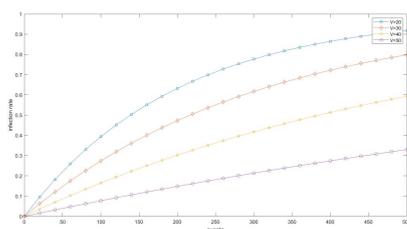


Figure 1. Size of the Space

According to Figure1, it can be figured out the relationship between the size of the room space and the infection rate. Compared with the line which represents the infection rate in a room of 10m³, the line represents the infection rate in a room of 50m³ is distinctively lower. While the quanta are about 500, the infection rate in a small room is three times higher than in a bigger room, in another word, the size of the space has a negative correlation with the infection rate.

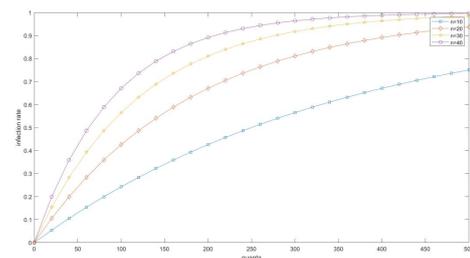


Figure 2. Ventilation Level of the Space

According to Figure2, the parameter n , which represents the ventilation time of a room in an hour, has a positive correlation with the infection rate. Under the condition when the quanta are 50, the probability of infection in a room ventilates ten times is nearly a half, while the probability of infection in a room ventilates ten times is lower than 10%. So we can come to the conclusion that is an infectious environment, it is important to ventilate in the space.

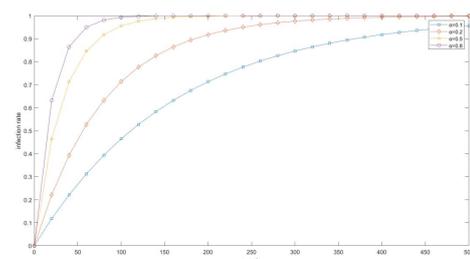


Figure 3. Co-location Time

According to Figure3, it could be figured out that the higher the co-location time is, the higher the probability of infection would appear. Under the condition when the quanta are 50, the probability of a person's infection whose co-location time takes up eighty percent of the whole period is over ninety percent, while the probability of a person's infection whose co-location time takes up only ten percent is about twenty percent. Here we can conclude that reducing co-location time would be a possible solution to reducing the infection rate.

To sum up, the three new additional parameters of the equation all reveal a correlation relationship with the probability of infection. According to the illustrations, we can find out that the size and ventilation of the space have a negative correlation with the infection rate, while the co-relation time has a positive correlation with the infection rate. Based on the outcoming, we can put forward a proposal that to avoid the possibility of being infected as much as possible, one should reduce time staying nearby an infected person and keep good ventilation condition in a big room.

4. Applying

4.1 Research Framework

Simulation and evaluation of passenger flow regulation using system dynamics model. The research framework is shown in Figure4.

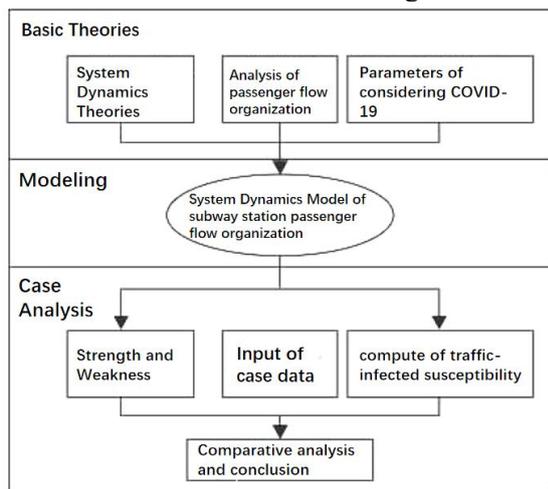


Figure 4. Research Framework

The system dynamics model of subway station passenger flow organization is established using system dynamics, passenger flow organization analysis, and control measures considering the epidemic situation. A variety of control measures are simulated according to the basic data, and the simulation results are used to analyze their impact on the dynamic change of subway station passenger flow and susceptibility evaluation parameters.

The passenger flow organization of subway stations can be roughly divided into inbound, outbound, and transfer processes. The specific process is shown in Figure5. This study simulates and selects four common control measures: limiting inbound passenger flow, controlling the number of service facilities,

extending the running streamline of station hall, and increasing the departure frequency of the subway.

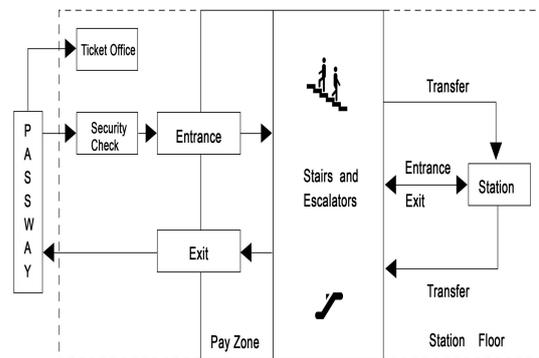


Figure 5. Passenger Flow Organization

4.2 Model Building

In this study, the analogic simulation platform is used to build the system dynamic (SD) model. The modeling elements of the system dynamics module include stock, flow, auxiliary variables, and constants. The system casual diagramme is shown in Figure6. By analyzing the causal relationship of the subway passenger flow organization system, the stock and flow diagram of the system is established by using the constituent elements to reflect the relationship between auxiliary variables and state variables in the system, the simulation of passenger flow organization is completed by quantifying the relationship between stock and flow chart.

The simulation area of the subway station passenger flow SD model is divided into station hall no paying area, station hall paying area, and platform layer. By analyzing the relevant variables affecting the passenger flow of the above three areas, the simulation area of the subway station is constructed Causality diagram.

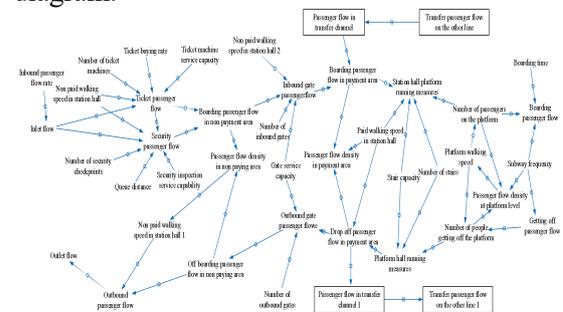


Figure 6. System Casual Diagram

4.3 System Dynamic Model

Considering the complexity of indoor

ventilation sources in subway station hall and platform public areas, the tunnel air brought by the entrance and exit of the station hall and the opening and closing of the platform screen door will affect the fresh air volume in public areas. The system flow diagram is shown in Figure 7. The definition of personnel flow in public areas is also different from the fixed-rated passenger capacity of vehicles. Therefore, the susceptibility calculation formula is expressed as

$$\frac{C}{S} = 1 - \exp\left(-\frac{\alpha I p q t / V}{V n_v}\right) \quad (11)$$

Where C represents the number of people who will have been newly infected; S represents the number of susceptible people in the space; p is the breath ventilation of susceptible individuals (m^3/h); Q is the constant rate of ventilation in an indoor environment (m^3/h); t represents the time when the susceptible individual is exposed in the environment (h); V is the volume of the room; I represents the number of people who have been infected; α is the time of exposure in an infectious environment is replaced by the co-location time. so this is the system flow diagram of modeling control measures:

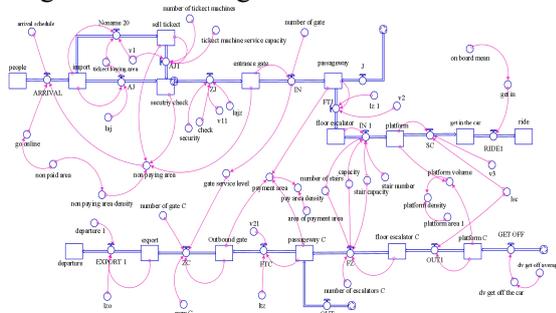


Figure 7. System Flow Diagram

To reduce the infection risk in the station, ensure the safety of passengers and staff, as well as the service level and quality in the station, reasonable passenger flow control measures, need to be taken. The control measures under the epidemic situation can be considered from the aspects of reducing the personnel density, increasing the personnel spacing, and reducing the residence time in the station; The traditional control measures mainly consider the traffic efficiency and avoid potential safety hazards caused by congestion when the bearing capacity in the station exceeds the maximum load.

5. Conclusions

Airborne diseases can transmit through air or droplets, which is especially infectious in indoor conditions. To find out the influence factors which may cause an infection, Wells put forward an equation to describe the rate of infection, and Riley developed the model into the Wells-Riley equation.

In our paper, we have built a model based on the Wells-Riley equation and applied the model to a realistic situation, while passengers get into the metro station hall, and use it to simulate the infection condition.

In this paper, we have built a probabilistic calculation model based on the Wells-Riley equation, applying the model to a realistic situation of a metro station. However, there are still some shortages of our work.

Firstly, in our assumption, we have assumed that the performance of the infection model changes linearly with ventilation rate, volume, and co-location time. In the real environment, the condition is more complicated and the model might not be suitable.

Secondly, we have mainly discussed the applying of our model in the station hall of a metro station. However, a complete trip of a passenger includes getting into the station, taking the subway, and leaving the station at the destination. We have not taken the rest part of a trip into account, which may affect the probability of infection.

In the future work, we plan to build a complete system dynamic model which contains the whole movement of a passenger's taking the metro, and figure out the infection rate based on the model.

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