Fuzzy Pricing of European Options Based on Constant Elasticity of Variance Process

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Abstract: Based on the assumption that the stock price follows the CEV process, this article uses fuzzy mathematics theory to discuss the price of European options. As the financial market is constantly fluctuating, the parameter of the stock price following the CEV process should not be a constant. Therefore, considering fuzzy interest rates, fuzzy stock prices, and fuzzy initial volatility, under the assumption of fuzzy parameters, the price of the obtained option is a fuzzy number. This article first derives the pricing formula for European options with stock prices following the CEV process. Then, using the theory of fuzzy mathematics, the fuzzy pricing formula for European options with stock prices following the CEV process is derived. In order to consider the membership degree of investors to a certain option price on a fuzzy number, the classic binary method is used to solve the membership degree. It indicates the value of options that investors or stock traders most hope to obtain, and finally uses fuzzy simulation algorithms in a fuzzy environment to calculate the expected value of options under different elasticity factors and iteration steps, and compares the membership degree of option value and make a simple analysis.

Keywords: Fuzzy Number; Dichotomy; Fuzzy Simulation; CEV Process; Fuzzy Random Variable; European Call Option

1. Introduction

In 1973, Black F and Scholes M deduced the closed form solution of European options, and the option pricing problem has been studied extensively, which also provides some theoretical basis for investors to make decisions. However, in the real stock market, the stock price does not obey geometric Brownian motion, which is inconsistent with the classical B-S pricing model. Later, many scholars improved the classical B-S pricing model, Cox [1] in 1975, the famous Constant elasticity of variance (CEV) model was derived, which is an option pricing model with random volatility. Schroder [2] then expanded the model by using non central chi square distribution to express CEV option pricing formula. However, the interest rate, initial volatility and initial stock price of the improved CEV model are all fixed real numbers, Therefore, the option pricing result is also a definite real number. In fact, in the CEV model, the value of interest rate, initial volatility and initial stock price should not be a fixed value, such as the risk-free interest rate r, which should fluctuate in the real market. Zadeh [3] put forward the theory of fuzzy sets in 1978, which can effectively represent uncertain parameter values. Through the theory of fuzzy sets, uncertain numbers can be changed into fuzzy numbers. Of course, the option price obtained is also a fuzzy number. Then Wu [4] designed an algorithm to transform the membership of an option price into an optimization problem, and solved it by dichotomy. Liu, YK [5] discussed the expected value of fuzzy random variables in continuous and discrete cases. This paper discussed the optimal value of option price in the CEV process based on dichotomy, and obtained the expected value of options through de fuzzification simulation, and analyze the relationship between the expected value and the optimal value.

2. Fuzzy Set Theory

Let U be a complete set, called mapping$\mu_{\tilde{A}}$: U $\rightarrow$[0, 1], x $\mapsto$ $\mu_{\tilde{A}}(x) \in [0,1]$ Determined the fuzzy subset on $\mu_{\tilde{A}}$, mapping $\mu_{\tilde{A}}$ Called $\tilde{A}$ The affiliation function of the $\mu_{\tilde{A}}(x)$ Called x pair $\tilde{A}$ The degree of affiliation.

Remark 1: From the definition it can be seen that the fuzzy subset consists of the affiliation function, the $\mu_{\widetilde{A}}$ uniquely determined,one can take the fuzzy subset that $\widetilde{A}$ and the affiliation function $\mu_{\widetilde{A}}$ The fuzzy subsets are referred to as fuzzy sets, and the degree of affiliation is referred to as the degree of affiliation.

Definition 2.1 [6] If $\forall\ \lambda \in [0,1]$, $\mu_{\widetilde{A}}(\lambda x + (1-\lambda)y) \geq min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$ A is called a convex fuzzy set.

Definition 2.2 $\widetilde{A}$ is a fuzzy subset, vs $\forall\ \alpha \in [0,1]$ Remember. $\widetilde{A}_\alpha = \{x: \mu_{\widetilde{A}}(x) \geq \alpha\}$ say/ call $\widetilde{A}_\alpha$ for $\widetilde{A}$ of α Intercept set, where α It is called a threshold or belief degree.

Definition 2.3 [7] $\widetilde{A}$ is called a fuzzy number if the following conditions are satisfied.

(1) $\widetilde{A}$ is a regular and convex fuzzy set; the

(2) $\widetilde{A}$ the affiliation function is upper semicontinuous.

(3) $\widetilde{A}$ of α The intercept set is bounded.

Remark 2 The affiliation function $\mu_{\widetilde{A}}$ Is upper semicontinuous at x1 if and only if $\forall \varepsilon > 0$,$\exists \delta > 0$"When $|x0 - x1| < \delta$ When, $\mu_{\widetilde{A}}(x0) < \mu_{\widetilde{A}}(x1) + \varepsilon$ By can be obtained if $\widetilde{A}$ is a convex fuzzy number,then its, the α level set $\widetilde{A}_\alpha$ is a convex set which is also a bounded closed set over the domain of real numbers, and can be denoted as $[\widetilde{A}_\alpha^L, \widetilde{A}_\alpha^U]$

Definition 2.4 $\widetilde{A}$ It is called an explicit number whose size is M if its membership function meets: $\mu_{\widetilde{A}}(x) = \begin{cases} 1 & x = M \\ 0 & Other \end{cases}$

Definition 2.5 $\widetilde{A}$ is a fuzzy set, and $\widetilde{A}_\alpha = \{x: \mu_{\widetilde{A}}(x) \geq \alpha\}$ it is α level set (cutoff set), then

$$\mu_{\widetilde{A}}(x) = \sup_{\alpha \in [0,1]} \alpha 1_{\widetilde{A}_\alpha}(x) \qquad (1)$$

The affiliation function of a triangular fuzzy number is defined as

$$\mu_{\widetilde{A}}(r) = \begin{cases} (r-a1)/(a2-a1) & a1 < r < a2 \\ (a3-r)/(a3-a2) & a2 < r < a3 \\ 0 & Other \end{cases}$$

Counterpart $\widetilde{A}$ of α The intercept can be expressed as,

$$\widetilde{A}_\alpha = [(1-\alpha)a1 + \alpha a2, (1-\alpha)a3 + \alpha a2]$$

Citation 1: If $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers, the $\widetilde{A} \oplus \widetilde{B}, \widetilde{A} \ominus \widetilde{B}, \widetilde{A} \oslash \widetilde{B}$, is a fuzzy number, and its α The level sets are as follows.

$$(\widetilde{A} \oplus \widetilde{B})_\alpha = [\widetilde{A}_\alpha^L + \widetilde{B}_\alpha^L, \widetilde{A}_\alpha^U + \widetilde{B}_\alpha^U]$$

$$(\widetilde{A} \ominus \widetilde{B})_\alpha = [\widetilde{A}_\alpha^L - \widetilde{B}_\alpha^U, \widetilde{A}_\alpha^U - \widetilde{B}_\alpha^L]$$

$$(\widetilde{A} \otimes \widetilde{B})_\alpha = [min\{\widetilde{A}_\alpha^L\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^L\widetilde{B}_\alpha^U, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^U\}, max\{\widetilde{A}_\alpha^L\widetilde{B}_\alpha^U, \widetilde{A}_\alpha^L\widetilde{B}_\alpha^U, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^U\}]$$

If $\widetilde{B}$ of α Horizontal set without 0,

$$(\widetilde{A} \oslash \widetilde{B})_\alpha = [min\{\widetilde{A}_\alpha^L\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^L\widetilde{B}_\alpha^U, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^U\}, max\{\widetilde{A}_\alpha^L\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^L\widetilde{B}_\alpha^U, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^L, \widetilde{A}_\alpha^U\widetilde{B}_\alpha^U\}]$$

Definition 2.6 If $\mathcal{F}$ is a real number $\mathbb{R}$ the set of all fuzzy subsets on $f(x): R \to R$ is a non-fuzzy function, the $\widetilde{A}$ is a fuzzy set, with, the $\tilde{f}: \mathcal{F} \to \mathcal{F}$ is a fuzzy-valued function, and, the $\tilde{f}(\widetilde{A})$ is a fuzzy subset and its affiliation function is defined by

$$\mu_{\tilde{f}(\widetilde{A})}(r) = \sup_{x: r = f(x)} \mu_{\widetilde{A}}(x) \qquad (2)$$

Lemma 2 If $\mathcal{F}$ is a real number $\mathbb{R}$ the set of all fuzzy subsets on $f(x): R \to R$ is a real-valued function, the $\widetilde{A}$ is a fuzzy set, with, the $\tilde{f}: \mathcal{F} \to \mathcal{F}$ is a fuzzy-valued function, and, the $\tilde{f}(\widetilde{A})$ is a fuzzy subset of, which α The level set can be expressed as follows.

$$(\tilde{f}(\widetilde{A}))_\alpha = \{f(X): x \in \widetilde{A}_\alpha\} \qquad (3)$$

Proof: on the one hand, if $r \in \{f(x): x \in \widetilde{A}_\alpha\}$, there is a point x that makes $r = f(x)$, $x \in \widetilde{A}_\alpha$ has the definition of a level set, the $\mu_{\widetilde{A}}(x) \geq \alpha$ Therefore. $\mu_{\tilde{f}(\widetilde{A})}(r) = \sup_{x: r = f(x)} \mu_{\widetilde{A}}(x) \geq \alpha$. $r \in (\tilde{f}(\widetilde{A}))_\alpha$, i.e $(\tilde{f}(\widetilde{A}))_\alpha \supseteq \{f(X): x \in \widetilde{A}_\alpha\}$ If, on the other hand $r \in (\tilde{f}(\widetilde{A}))_\alpha$. And then $\sup_{x: r = f(x)} \mu_{\widetilde{A}}(x) \geq \alpha$ That is, there is a point x that makes, $\mu_{\widetilde{A}}(x) \geq \alpha$ As well as $r = f(x)$ So it can be obtained that $r \in \{f(x): x \in \widetilde{A}_\alpha\}$.

3. Fuzzy Formula under CEV Process

In the CEV model, it is assumed that the stock price follows the following process:

$$dS_t = rS_t dt + \sigma_0 S_t^{\frac{\beta}{2}} dB_t \qquad (4)$$

Among them $r \geq 0$ is a constant, the risk-free rate. $\sigma_0 > 0$ As well as $\beta \in [0,2]$ is a parameter of the model for that $\beta > 2$ Emanuel [8] first proposed, and then extended Schroder to the expression form of non central chi square distribution.

This paper only considers $\beta \in [0,2]$ the situation. Obviously when, the $\beta = 2$ when the price of the stock obeys geometric Brownian motion $dS_t = rS_t dt + \sigma_0 S_t dB_t$, parameters β. The selection of the model will make the model more consistent with the actual financial market. Definition $x = S^{2-\beta}$, from Ito's Lemma it follows that.

Before giving the analytical expression of CEV model, we must first give its transition probability density function.

$$dx_t = x^{'}(S)dS + \frac{1}{2}x^{''}(S)dt$$
$$= (2-\beta)S^{1-\beta}(rS_t dt + \sigma_0 S^{\frac{\beta}{2}} dB_t) + \frac{1}{2}(x^{'}(S)dS)^{'}(S)dS$$
$$= (2-\beta)rx_t dt + (2-\beta)\sigma_0\sqrt{x_t}dB_t + \frac{1}{2}(2-\beta)(1-\beta)\sigma_0^2 dt$$
$$= (2-\beta)[rx_t + \frac{1}{2}(1-\beta)\sigma_0^2]dt + (2-\beta)\sigma_0\sqrt{x_t}dB_t$$

Lemma3: For a general stochastic process $dx_t = \mu(x_t,t)dt + \sigma(x_t,t)dB_t$ Its Fokker-Planck equation can be expressed as follows.

$$\frac{\partial P}{\partial t} = \frac{1}{2}\frac{\partial(\sigma^2 P)}{\partial x^2} - \frac{\partial(\mu P)}{\partial x}$$

Prove that such that $g = g(x_t)$, from Ito's Lemma it follows that.

$$dg = g^{'}(x)dx + \frac{1}{2}g^{''}(x)dt$$
$$= \frac{\partial g}{\partial x}(\mu dt + \sigma dB_t) + \frac{1}{2}\sigma^2\frac{\partial^2 g}{\partial x^2}dt$$
$$= [\frac{1}{2}\sigma^2\frac{\partial^2 g}{\partial x^2} + \mu\frac{\partial g}{\partial x}]dt + \sigma\frac{\partial g}{\partial x}dB_t$$

The expectation is taken from both sides of the equation, which is 0 after the last term of the property of Brownian motion is taken as the expectation, and dt is not 0, which is obtained by moving to the left of the equation

$$\frac{dE(g)}{dt} = E(\mu\frac{\partial g}{\partial x}) + E(\frac{1}{2}\sigma^2\frac{\partial^2 g}{\partial x^2}) \quad (5)$$

Make $g(x_t)$ The probability density function at time t is $P(x,t)$So $g(x_t)$ The expectation is that the

$E(g(x_t)) = \int g(x)P(x,t)dx$

namely

$$E(\frac{dgx_t}{dt}) = \int g(x)\frac{\partial P}{\partial t}dx \quad (6)$$

From Equations 5 and 6 we obtain.

$$\int g\frac{\partial P}{\partial t}dx = \int[\mu\frac{\partial P}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 g}{\partial x^2}]Pdx$$

And because

$$\int \mu\frac{\partial g}{\partial x}Pdx = -\int g\frac{\partial(\mu P)}{\partial x}dx$$
$$\int \sigma^2\frac{\partial^2 g}{\partial x^2}Pdx = \int g\frac{\partial(\sigma^2 P)}{\partial x^2}Pdx$$

So

$$\int g[\frac{\partial P}{\partial t} + \frac{\partial(\mu P)}{\partial x} - \frac{1}{2}\frac{\partial(\sigma^2 P)}{\partial x^2}]dx = 0$$

So

$$\frac{\partial P}{\partial t} = \frac{1}{2}\frac{\partial(\sigma^2 P)}{\partial x^2} - \frac{\partial(\mu P)}{\partial x}$$

The F-P formula given above is solved by using the Feiler lemma . First, a parabolic equation satisfying $(P)_t = (axP)_{xx} - ((bx+h)P)_x$.

Lemma4 Order $P(x,t|x0)$ Is for x, t at x_0 conditions, then its probability density function, i.e., the solution of the parabolic equation is denoted by

$$P(t,x|x_0) = \frac{b}{a(e^{bt}-1)}(\frac{e^{-btx}}{x_0})^{(h-a)/2a}$$
$$exp[\frac{-b(x+x_0e^{bt})}{a(e^{bt}-1)}]I_{1-h/a}(\frac{2b}{a(1-e^{-bt})}(e^{-bt}xx_0)^{1/2})$$

Among them $I_k(x)$ Is the Bessel function of order k of the first kind, expressed as

$$I_k(x) = \sum_{r=0}^{\infty}\frac{(x/2)^{2r+k}}{\Gamma(r+1+k)}$$

Therefore $P(S_T,T|S_t,T>t) = P(x_T,T|x_t,T>t|M)$, M means $M = (2-\beta)S^{1-\beta}$By Lemma 4, it follows that

$$a = (1/2)\sigma_0^2(2-\beta)^2$$
$$b = r(2-\beta)$$
$$h = (1/2)\sigma_0^2(2-\beta)(1-\beta)$$

Having.

$$P(S_T,T|S_t,T>t) = (2-\beta)k^{\frac{1}{2-\beta}}(yw^{1-2\beta})^{\frac{1}{2(2-\beta)}}e^{-y-w}I_{1/(2-\beta)}(2\sqrt{yw})$$

Where k, y and w are respectively:

$$k = \frac{2r}{\sigma_0^2(2-\beta)[e^{r(2-\beta)T}-1]}$$
$$y = kS_t^{2-\beta}e^{r(2-\beta)T} \quad (7)$$
$$w = kS_T^{2-\beta}$$

The option pricing formula is obtained through the non central chi square distribution. The pricing formula of the reference Schroder is combined with the probability density function, so the pricing formula of the CEV model is expressed as

C= $E(max(0,S_T-K))$

$$= e^{-r\tau}\int_K^{\infty} S_T P(S_T,T|S_t,T>t)dS_T - e^{-r\tau}\int_K^{\infty} K P(S_T,T|S_t,T>t)dS_T$$
$$= e^{-r\tau}\int_K^{\infty} P(S_T,T|S_t,T>t)(S_T-K)dS_T$$
$$= C_1 - C_2$$

Among them $\tau = T - t$ Because $w = kS_T^{2-\beta}$, which is obtained by deducing that dS_T and dw Relationships.

$$w^{2-\beta} = k^{2-\beta} S_T$$
$$S_T = k^{2-\beta} w^{1/(2-\beta)}$$
$$dS_T = (2-\beta)^{-1} k^{-1/(2-\beta)} w^{\beta-1/(2-\beta)} dw$$

For, the $w = kS_T^{2-\beta}$ make

$$z = kK^{2-\beta}$$

Let the value of w start from z, so C1 can be calculated

$$C1 = e^{-r\tau}\int_z^\infty e^{-y-w}(y/w)^{1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw}(w/k)^{1/(2-\beta)}dw$$
$$= e^{-r\tau}\int_z^\infty e^{-y-w}(y/w)^{1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw})(w/y)^{1/(2-\beta)}(y/k)^{1/(2-\beta)}\mathrm{dw}$$
$$= e^{-r\tau}(y/k)^{1/(2-\beta)}\int_z^\infty e^{-y-w}(y/w)^{-1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$
$$= e^{-r\tau}S_t e^{r\tau}\int_z^\infty e^{-y-w}(y/w)^{1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$
$$= S_t\int_z^\infty e^{-y-w}(y/w)^{1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$

$$C2 = Ke^{-r\tau}\int_z^\infty (2-\beta)k^{1/(2-\beta)}(yw^{1-2\beta})^{1/(4-2\beta)} e^{-y-w} I_{1/(2-\beta)} \left(2\sqrt{yw=}\right)\frac{k^{-1/(2-\beta)}}{2-\beta} w^{(\beta-1/2-\beta)}dw$$
$$= Ke^{-r\tau}\int_z^\infty y^{1/(4-2\beta)} w^{-1/(4-2\beta)} e^{-y-w} I_{1/(2-\beta)}(2\sqrt{yw})dw$$
$$= Ke^{-r\tau}\int_z^\infty e^{-y-w}(y/w)^{1/(4-\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$

Similarly, C2 can also be calculated

By the degree of freedom of the non-central cartesian distribution, the ν, the non-central parameters are λ The probability density function of is denoted as

$$f_{\chi_\nu^2(\lambda)}(y) = \frac{1}{2}\left(\frac{y}{\lambda}\right)^{(\nu-2)/4} I_{(\nu-2)/2}(\sqrt{\lambda x})e^{-(\lambda+y)/2} = f(y,\nu,\lambda)$$

By the definition of distribution function and probability density function, the $Q(m,\nu,\lambda) = \int_m^\infty f(l,\nu,\lambda)$ make $\overline{w} = 2w$ and $\overline{x} = 2x$, C1 is expressed as

$$C = S_t\int_z^\infty e^{-y-w}(y/w)^{1/(4-2\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$
$$= S_t\int_{2z}^\infty e^{-(\bar{w}+\bar{y})/2}\left(\frac{\bar{w}}{\bar{y}}\right)^{1/(4-2\beta)} I_{1/(2-\beta)}\frac{1}{2}(2\sqrt{\bar{y}\bar{w}})d\bar{w}$$
$$= S_t Q(2z,\nu,\overline{x})$$
$$= S_t Q(2z, 2+2/(2-\beta), 2y)$$

Because $(\nu-2)/2 = 1/(2-\beta)$ And so it is $\nu = 2+2/(2-\beta)$ And there is $I_k(x) = I_{-k}(x)$ So

$$C2 = Ke^{-r\tau}\int_z^\infty e^{-y-w}(y/w)^{1/(4-\beta)} I_{1/(2-\beta)}(2\sqrt{yw})dw$$
$$= Ke^{-r\tau}\int_{2z}^\infty e^{-(\bar{y}+\bar{w})/2}\left(\frac{\bar{y}}{\bar{w}}\right)^{1/(4-\beta)} I_{\frac{1}{2-\beta}}(2\sqrt{\bar{y}\bar{w}}\frac{1}{2})d\bar{w}$$
$$= Ke^{-r\tau}Q(2z, 2-2/(2-\beta), 2y)$$

Because

$$1 - Q(2l, 2\nu-2, 2\lambda) = \int_m^\infty f(2l, 2\nu, 2\lambda)$$

So

$$C2 = Ke^{-r\tau}\int_z^\infty P(2y, 2+2/(2-\beta), 2w)dw$$
$$= Ke^{-r\tau}\int_z^\infty Q(2z, 2+2/(2-\beta), 2y)$$
$$= Ke^{-r\tau}(1 - Q(2y, 2+2/(2-\beta), 2z))$$

so

$$C = S_t Q(2z,\nu,\overline{x}) - Ke^{-r\tau}(1 - Q(2y, 2+2/(2-\beta), 2z))$$

At the moment $t = 0$ The pricing formula for time rights is expressed as follows.

$$C(S_0) = S_0 Q(2z, 2/(2-\beta), 2y) - Ke^{-rT}[1 - Q(2y, 2/(2-\beta), 2z)]$$

The parity formula of options is obtained from Musiela [9]

$$C_t - P_t = S_t - Ke^{-r(T-t)} \quad (8)$$

The pricing formula for the put option is obtained as

$$P(S_0) = Ke^{-rT}Q(2y, 2/(2-\beta), 2z) - S_0[1 - Q(2z, 2/(2-\beta), 2y)]$$

4. The Fuzzy Formula

The pricing formula for the European option is obtained as, the

$$C(S_0) = S_0 Q(2z, 2/(2-\beta), 2y) - Ke^{-rT}[1 - Q(2y, 2/(2-\beta), 2z)$$

Among them $r \geq 0$ is a constant, the risk-free rate. $T > 0$ denotes the time at which the option expires as well as, the $\beta \in [0,2]$ Is the parameter of the model, K is the strike price of the option, S_0 is the price of the stock at the initial moment. The parameters inside the non-centered chi-square distribution are, respectively, the $k = \frac{2r}{\sigma_0^2(2-\beta)[e^{r(2-\beta)T}-1]}$, $y = kS_t^{2-\beta}e^{r(2-\beta)T}$, $z = kK^{2-\beta}$ In the previous assumption, r, σ_0, as well as S_0 are a constant, now assume that they are all fuzzy numbers respectively $\tilde{r}$, $\widetilde{\sigma_0}$, $\widetilde{S_0}$, so k, y, z are also fuzzy numbers, respectively,

$$\tilde{k} = \frac{2\tilde{r}}{\widetilde{\sigma_0}^2(2-\beta)[e^{\tilde{r}(2-\beta)T}-1]}$$

$$\tilde{y} = \tilde{k}\widetilde{S_0}^{2-\beta}e^{\tilde{r}(2-\beta)T}$$

$$\tilde{z} = \tilde{k}K^{2-\beta}$$

The pricing formula for a fuzzy European option can now be expressed as, the

$$\widetilde{C_0} = \widetilde{S_0}\tilde{Q}(2\tilde{z}, 2 + 2/(2-\beta), 2\tilde{y}) - Ke^{-\tilde{r}T}[1 - \tilde{Q}(2\tilde{y}, 2/(2-\beta), 2\tilde{z}) \quad (9)$$

To express the degree of affiliation, it is first necessary to express the option price fuzzy number to get α The pricing formula obtained from the non central chi square distribution of the level set is obviously not a function that increases or decreases with the parameters y and z, so it is not easy to pass $\tilde{k}, \tilde{y}, \tilde{z}$ of α level sets giving their option prices α Horizontal sets. But by the expansion principle, it is possible to give its α of the expression for the level set, such that $\widetilde{C}_\alpha = [C_1(\alpha), C_2(\alpha)]$, C_1 and C_2 said α The minimum and maximum values of the level set at the time of affiliation. Let, the $F(S_0, T, r, K, \sigma_0) = C_{S_0}$ Then you can get.

$$\begin{aligned} C_1(\alpha) &= \min\{F(S_0, E, r, \sigma_0, T)|A\} \\ C_2(\alpha) &= \max\{F(S_0, E, r, \sigma_0, T)|A\} \end{aligned} \quad (10)$$

Where A represents $S_0 \in [S_1(\alpha), S_2(\alpha)]$, $r \in [r_1(\alpha), r_2(\alpha)]$, $\sigma_0 \in [\sigma_1(\alpha), \sigma_2(\alpha)]$. $[S_1(\alpha), S_2(\alpha)]$,$[r_1(\alpha), r_2(\alpha)]$,$[\sigma_1(\alpha), \sigma_2(\alpha)]$ are fuzzy numbers, respectively $\widetilde{S_0}$, $\tilde{r}$, $\widetilde{\sigma_0}$ of α Level Set.

4.1 AlgoritSSes

Assume that C is the fuzzy price of European call options $\widetilde{C_t}$ For a value at time t, the membership degree of its value C, that is, a belief degree of investors, can be solved by α Convert to the following optimization problem.

$$\max \quad \alpha$$
$$s.t. \quad (\widetilde{C_t})_\alpha^L \leq C \leq (\widetilde{C_t})_\alpha^U \text{ , } 0 \leq \alpha \leq 1$$

Make $g(\alpha) = (\widetilde{C_t})_\alpha^L$ As well as $h(\alpha) = (\widetilde{C_t})_\alpha^U$ "When $C_t < h(1)$ The abbreviations are as follows.

Step 1.ε It's precision.α_0 is the initial value.

Step 2 $\alpha \leftarrow \alpha_0$

Step 3 $L \leftarrow 0$

Step 4 $U \leftarrow 1$

Step 5 $g(\alpha) = (\widetilde{C_t})_\alpha^{min}$ If $g(\alpha) \leq C$ Go to step 6, otherwise go to step 7. Go to step 6, otherwise go to step 7.

Step 6 $C - g(\alpha) < \varepsilon$ Then the end of the program results in α Otherwise, let $L \leftarrow \alpha$, $\alpha \leftarrow (L+U)/2$ Then go to step 5. Then go to step 5.

Step 7 $U \leftarrow \alpha$, $\alpha \leftarrow (L+U)/2$ Then go to step 5.

4.2 Fuzzy Simulation Algorithms

By Reference [10],the fuzzy simulation algorithm is abbreviated as follows, the ξ is a fuzzy random variable.

The first step is to uniformly generate N points $u_i = X(\theta_i)$ From affiliation μ_X , $i = 1,2,3 \ldots, N$.

Step 2 Calculate $e_i = E[f(\xi(\theta_i))]$.

Step 3 For μ_i, e_i Reordering such that $e_1 \leq e_2 \leq \ldots \leq e_N$, $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_N$.

Step 4 Calculate the weights from the weighting formula p_i .

Step 5 Calculate the fuzzy expectation that $E(f(\xi)) = \sum_{i=1}^{N} e_i p_i$

5. Numerical Simulations

Assuming that E=50,T=1,σ_0=(0.28,0.3,0.32), r(0.1,0.2,0.3),S_0(48,50,52), Then the corresponding option price fuzzy number can be obtained, and the parameters of the benchmark model are shown in Table 1.

Table 1. Parameters of the Baseline Model

parameters	Symbol	parameters	Symbol
Finalizing the price	*E*=50	fuzzy stock prices	*S*(48,50,52)
Initial time	*t*=0	fuzzy initial volatility	σ_0(0.28,0.3,0.32)
β	β ∈[0,2]	fuzzy interest rates	*r*(0.1,0.2,0.3)

5.1 Simulate Option Price Ranges with Different Degree of Belief

By Algorithm 1, if the price of the option, the $C \geq h(1)$ Then the Algorithm 1 restriction reads $-h(\alpha) \leq - C$ The same applies, so the degree of belief in some option prices can be obtained , as shown in Table 2.

Table 2. Different Option Belief Degree

C	10.16	9.12	8.86	8.46	7.76	5.76
α	0.546	0.998	0.966	0.913	0.857	0.535

Assuming that the price of the option is 8.86, the degree of confidence obtained through the calculation of Algorithm 1 is 0.966, if this degree of confidence is acceptable to investors or financial analysts, then it can also be used as a reference basis for decision-making. In fact $S_0 = 50, r = 0.2, \sigma_0 = 0.3, \beta = 1$ The price of the option is 9.13, at which point the belief degree is 1. It is also possible to obtain some affiliation with the α The level sets, as shown in the Table 3 below.

Table 3. Option Price Ranges

C	(9.10, 9.16)	(8.45,9.26)	(7.83, 10.27)	(5.96, 11.56)	(2.83, 14.48)
α	0.99	0.9	0.8	0.5	0

when $\alpha = 0.9$ The value of the European call option is in the interval [8.45,9.26] between the belief degree of 0.9, indicating that investors or financial analysts for (belief degree) affiliation degree of 0.9 is acceptable, then you can for the interval [8.45,9.26] for any of the value for its investment decision-making reference.

5.2 Fuzzy Simulation of Option Prices

Hypothesis $\xi = (\widetilde{\sigma_0}, \tilde{r}, \widetilde{S_0})$ denote a fuzzy variable whose affiliation function can be expressed as, the

$$\mu_\xi(u) = \min_{1 \leq i \leq 5} \mu^i(u)$$

Among them $\mu^1 = \mu_{\sigma_0}$, $\mu^2 = \mu_r$, $\mu^3 = \mu_{S_0}$. Thebenchmark parameters are shown in Table 1, so from Algorithm 2 we can get the simulated price of the option relative to the fuzzy path as shown in Table 4 and Figure 1.

Table 4. Different under the Baseline Parameters α Convergence Value

	100	500	1000	2000	3000	5000
0	5.1838	5.4412	5.5964	5.7788	5.8155	5.9706
1	5.2820	5.4161	5.5198	5.8345	5.9746	6.0193
2	5.4597	5.7342	5.8213	5.9014	5.9155	6.1236

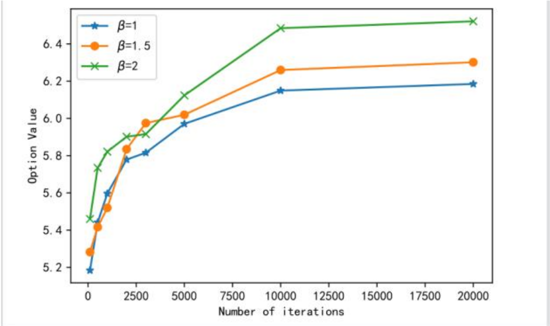

Figure 1. Expected Value of the Simulated Option

6. Conclusion

This paper studies the problem of fuzzy option pricing in the CEV process. Because of the volatility of the financial market at the moment, even in the CEV process, the interest rate, the initial stock price, and the initial volatility should not be constants, the uncertainty of this parameter can be effectively expressed by the theory of fuzzy sets. Through the dichotomy algorithm, a degree of confidence (membership) of different option prices can be obtained for financial analysis or investors' reference, the price range of options with different membership degrees is calculated. The parameter r is analyzed, σ_0, K, S_0. The larger the K is, the lower the option price is, σ_0 ,K, S_0 The larger the option price is the higher the option price is the same as the real market situation, through the fuzzy simulation method to get the expected value of fuzzy option price, to verify the dichotomous algorithm of a reliability. It can be seen that different elasticity coefficients (α) the option prices are all converging to a converging value vs. the option price with an affiliation of 0.5 (5.96) suggests that theoretically the option prices in the financial market are basically in the neighborhood of the option price with a belief degree of 0.5.

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