A method for Designing a Guidance Law Utilizing Linear Quadratic Forms based on Differential Games in 3-Dimensional Space

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Abstract: Consider a T-A-D game scenario in space: the target strives to evade the attacker, the defender intercepts the target, and the attacker must both avoid the target's counterattack and attempt to hit the target. It is assumed that all entities are intelligent, anticipating the next state of their adversaries and making the most suitable maneuvering strategy based on this prediction. This paper presents a 3-dimensional linear-quadratic differential game guidance law for target-attacker-defender scenarios, which is more intelligent than traditional guidance laws and is suitable for future intelligent environments.

Keywords: Differential Game; Linear-Quadratic; Target-Attacker-Defender

1. Introduction

The theory of differential games was first introduced by Isaacs in 1966 [1]. Subsequent scholars have extensively discussed and studied this theory in pursuit-evasion problems (or their derivatives) [2], and it has been successfully applied in the field of missile guidance [3]. Over the years, the main contributions of differential game guidance laws in the field of air combat are as follows:

Pursuit-evasion models are categorized based on the number of pursuers and evaders into one-pursuer-one-evader (1v1), N-pursuers-one-evader (Nv1), one-pursuer-M-evaders (1vM), and N-pursuer-M-evaders (NvM) [4].

The application of pursuit-evasion models in the field of air combat began relatively early. The earliest missile pursuit-evasion games were established based on aircraft pursuit-evasion models. Due to the limitations of computational power and theory, the aircraft were treated as mass points moving at a constant speed within a plane, with trajectories considered as segmented curves [5]. In 1980, Shinar modeled the missile-aircraft pursuit-evasion problem, establishing a two-body dynamic model, setting initial missile-aircraft orientations, and extending the pursuit-evasion from two dimensions to T-A-D dimensions [6]. In 1987, Moritz [7] introduced constraints such as ballistic inclination and velocity inclination limits when establishing a missile two-body pursuit-evasion model. Since then, scholars have focused their research on two main aspects: making the models more realistic and improving the accuracy of existing models.

In terms of establishing models based on real combat scenarios, most studies treat the missile as a mass point. Under the 1v1 framework, the focus is mainly on establishing more realistic state variable constraints, such as constraints on the missile's turning radius and maximum lift [8], velocity constraints [9], and lateral acceleration constraints [10]. Based on this, the two-body pursuit-evasion problem is extended to two two-body pursuit-evasion problems, i.e., the T-A-D confrontation model [11, 12]. Under the Nv1, Mv1, and MvN frameworks, scholars have proposed various boundary value constraints, such as pursuit area constraints [13], simultaneous arrival constraints for N pursuers [14], and area coverage constraints [15].

The subsequent arrangement of this paper is as follows: The second section models the target-attacker-defender (T-A-D) game; the third section derives the differential game guidance law for the T-A-D game using the linear-quadratic differential game method; the fourth section simulates and analyzes based on this guidance method; and the fifth section presents the conclusions.

2. T-A-D Game Kinematic Modeling

2.1 Assumptions

Given that 3-dimensional motion can be decomposed into two orthogonal planar motions, this study focuses solely on the motion within the pitch plane and makes certain idealized assumptions:

(1) The relative motion trajectories of the attacker with respect to the target and the defender with respect to the attacker can be linearized near the initial target line of sight;

(2) The attacker, target, and defender are all considered as point masses;

(3) Each entity has access to precise information about each other;

(4) The velocity and maximum aerodynamic load of the attacker and defender during the terminal guidance phase are constant.

2.2 Nonlinear Kinematic Model

The positional relationship and parameter description of the target, attacker, and defender are shown in Figure 1.

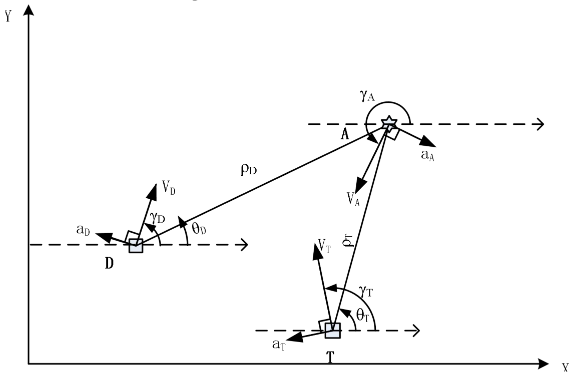

Figure 1. Relative Motion Relationship of Target, Attacker, and Defender within the Plane

The relative motion of the target T with respect to the attacker A:

$$\dot{\rho}_T = V_T \cos\left(\gamma_T - \theta_T\right) - V_A \cos\left(\gamma_A - \theta_T\right) \quad (1)$$

$$\rho_T \dot{\theta}_T = V_T \sin\left(\gamma_T - \theta_T\right) - V_A \sin\left(\gamma_A - \theta_T\right) \quad (2)$$

The relative motion of the defender D with respect to the attacker A:

$$\dot{\rho}_D = V_A \cos\left(\gamma_A - \theta_D\right) - V_D \cos\left(\gamma_D - \theta_D\right) \quad (3)$$

$$\rho_D \dot{\theta}_D = V_A \sin\left(\gamma_A - \theta_D\right) - V_D \sin\left(\gamma_D - \theta_D\right) \quad (4)$$

In the final stage of engagement, it is assumed that the velocities of the attacker, defender, and target are constant. The remaining flight time of the attacker A to the target T near the collision triangle can be expressed as follows:

$$t_T = \frac{\rho_T}{-\dot{\rho}_T} \quad (5)$$

where ρ_T is the initial distance and $-\dot{\rho}_D$ is the approach velocity.

Similarly, the remaining flight time of the defender D to the attacker A:

$$t_D = \frac{\rho_D}{-\dot{\rho}_D} \quad (6)$$

where ρ_D is the initial distance and $-\dot{\rho}_D$ is the approach velocity.

If the control lag of the autopilot is not considered, then:

$$\dot{\gamma}_A = \frac{a_A}{V_A} \quad (7)$$

$$\dot{\gamma}_D = \frac{a_D}{V_D} \quad (8)$$

$$\dot{\gamma}_T = \frac{a_T}{V_T} \quad (9)$$

2.3 Linear Kinematic Model

Linearizing the aforementioned nonlinear motion model near the collision triangle, the assumptions are:

$$x_1 = [\rho_T \quad \dot{\rho}_T \quad a_A \quad a_T] \quad (10)$$

$$x_2 = [\rho_D \quad \dot{\rho}_D \quad a_A \quad a_D] \quad (11)$$

The state-space equations for the T-A-D game are then:

$$\dot{x}_1 = A_1 x_1 + B_1 [a_A \quad a_T] \quad (12)$$

$$\dot{x}_2 = A_2 x_2 + B_2 [a_A \quad a_D] \quad (13)$$

where,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_A & a_T \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_A & a_D \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Introducing the zero effort miss (ZEM) method to reduce the order of the above model and predict the terminal miss distance for this state:

$$z_1 = L_1 \Phi_1 x_1 \quad (14)$$

$$z_2 = L_2 \Phi_2 x_2 \tag{15}$$

where z_1 and z_2 are the zero effort miss distances for the attacker with respect to the target and the defender with respect to the attacker, respectively.

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_1 = L^{-1}\left[\left(sI - A_1\right)^{-1}\right]^T$$

$$\Phi_2 = L^{-1}\left[\left(sI - A_2\right)^{-1}\right]^T$$

3. Linear-Quadratic Differential Game Guidance Law

Based on Section 2, considering the ZEM and fuel consumption of the individual, the linear-quadratic performance function can be formulated as follows:

$$\begin{aligned} J_{(a_A, a_T, a_D)} &= |z_1| - \varepsilon_1 |z_2| + \frac{1}{2}\varepsilon_2 \int_0^{t_T} a_A{}^2 dt \\ &- \frac{1}{2}\varepsilon_3 \int_0^{t_T} a_T{}^2 dt \\ &- \frac{1}{2}\varepsilon_4 \int_0^{t_D} a_D{}^2 dt \end{aligned} \tag{16}$$

where ε_1 ε_2 ε_3 ε_4 is the game regulation coefficient, which can adjust the conflict in maneuvering strategies of the attacker A when facing the target T and the defender D. Considering the differential game scenario, the implicit form of the differential game guidance law can be derived:

$$\begin{cases} a_D^* = \underset{a_D, a_T, a_A^*}{\arg\min} (J) \\ a_T^* = \underset{a_D, a_T, a_A^*}{\arg\min} (J) \\ a_A^* = \underset{a_D^*, a_T^*, a_A}{\arg\max} (J) \end{cases} \tag{17}$$

4. Simulations

Using the differential game guidance law derived above, simulations of the T-A-D game are conducted with the initial states and parameters as shown in Table 1.

Table 1. Initial States of Target-Attacker-Defender

Parameter	T	A	D
Initial Position (km)	(0,0,0)	(10,8,10)	(-5,-8,-8)
Initial Velocity (m/s)	500	1000	1000
Initial Acceleration (m/s^2)	0	0	0
Simulation Time (s)	25	25	25
Time Step (s)	0.001	0.001	0.001
Regulation Coefficient	$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$		

The spatial trajectories of the T-A-D game, obtained from the initial states and parameters set in Table 1, are shown in Figures 2-4. Here, the vertical upward direction represents the positive Y-axis, and the XYZ axes are determined according to the right-hand rule coordinate system. Figure 2 illustrates the X-Z cross-sectional trajectory of the T-A-D three-body game; Figure 3 shows the Y-Z cross-sectional trajectory; and Figure 4 represents the X-Y cross-sectional trajectory.

From the figures, it can be observed that the 3-dimensional spatial T-A-D game model and method based on the differential game guidance law are effective and have certain reference value for engineering practice and similar fields, such as UAV confrontation.

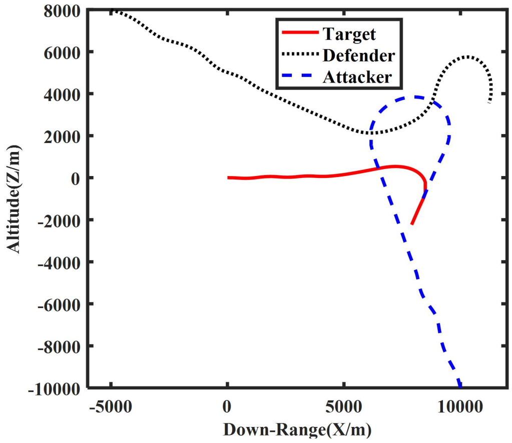

Figure 2. X-Z Cross-Sectional Trajectory of the T-A-D Game

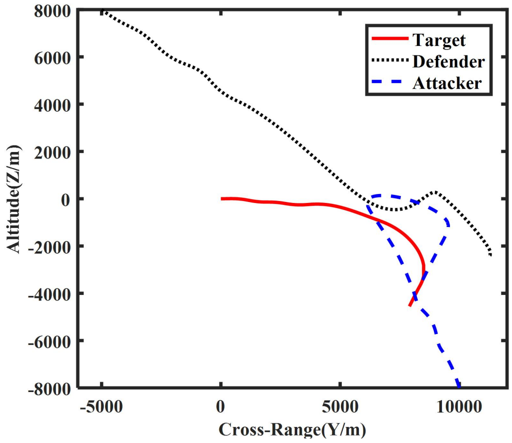

Figure 3. Y-Z Cross-Sectional Trajectory of the T-A-D Game

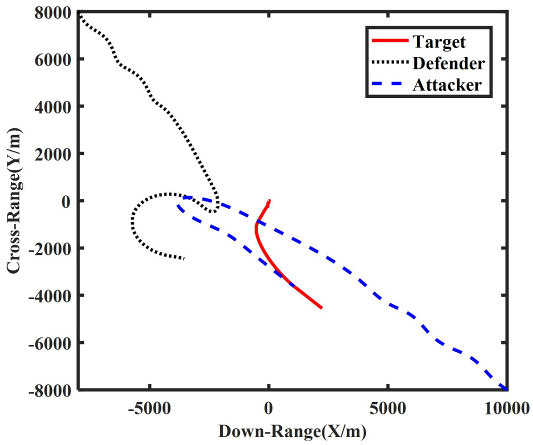

Figure 4. X-Y Cross-Sectional Trajectory of the T-A-D Game

5. Conclusion

This paper establishes a 3-dimensional relative kinematic model for the T-A-D game scenario and performs linearization and order reduction of the mathematical model based on the zero effort miss (ZEM) method, which simplifies the subsequent derivation of the guidance law. An optimized model of the linear-quadratic differential game guidance law is presented using the differential game method, and the differential game guidance law is derived. Simulation analysis based on this guidance law indicates that the designed differential game guidance law is effective and can be further considered for application and improvement in other fields.

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