Optimization of Carbon Board Design Based on Multi-day Standard Optimization Model

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Abstract: As people become increasingly aware of the potential of carbon board technology to significantly improve athlete performance, especially in long-distance events, this study aims to address limitations of existing carbon board optimization research. Carbon board running shoes are added to the original running shoes. Carbon board is a carbon fiber composite material. It has the advantages of light weight, toughness, more elasticity, and good fatigue resistance. Compared with soft soles, carbon board running shoes can improve the support for the arch of the foot, making the support more uniform, which in turn can improve exercise ability and reduce the possibility of injury. In this paper, finite element analysis, biomechanics principles and multi-objective optimization strategies are used to systematically study the geometric, mechanical and performance characteristics of carbon boards. Key factors such as carbon plate shape, size, weight, elasticity and thickness are optimized to improve shock absorption, rebound performance and overall sports performance. The optimized carbon board design, with a length of 0.1672 m, a width of 0.0119 m, and a thickness of 0.0079 m, achieves a delicate balance between structural rigidity, energy conversion efficiency and the use of lightweight materials. The results of this research provide a solid foundation for the innovative design of carbon-board running shoes, improving athletic performance and shoe comfort.

Keywords: Carbon-plate Running Shoes; Optimize Carbon Plate; Finite Element; Biomechanics Principles; Multi-objective Analysis

1. Introduction

In various sports activities, the sole of running shoes serves as the primary and direct medium for runners to interact with the track, with the ground reaction forces reaching several times the weight of the runner [1]. In the design of professional sports equipment, carbon plate technology, which utilizes carbon fiber-reinforced materials, has become a decisive element in enhancing athletes' performance due to its exceptional lightweight and elasticity. Carbon plates, made of carbon fiber-reinforced epoxy resin, were first applied to basketball shoes in the early 1990s, significantly improving athletes' foot support and torsional resistance. Technological innovation has led to a leap in the performance of running shoes, especially with the advent of carbon plate running shoes, which has brought revolutionary impacts to the field of sports. Take the outstanding marathon runner Eliud Kipchoge as an example; he broke the long-distance running world records several times in the 2019 competition with the help of carbon plate running shoes, highlighting the significant role of this material in enhancing sports performance and stimulating academic exploration of its underlying mechanisms [2].

In the contemporary field of sports shoe design, carbon plate technology has become a highly scrutinized research focus, and its potential to enhance athletes' performance has gained widespread recognition and in-depth investigation. The main idea of carbon plate running shoes (Figure 1) is: a thicker elastic midsole + full palm arc-shaped propulsion carbon plate + higher front palm height + overall lightweight [3].

This design concept is not only based on the careful collection and analysis of sports data but also reveals the key elements of performance enhancement through advanced data processing technology, which is then transformed into innovative sports shoe design. This design strategy particularly emphasizes providing athletes with strong propulsion during competitions, thereby significantly improving their competitive achievements. Moreover, by meticulously adjusting the shape and response characteristics of the carbon plate and combining it with high-performance cushioning materials, it further optimizes the energy conversion and rebound performance during running, bringing a smoother and more efficient running experience to athletes.

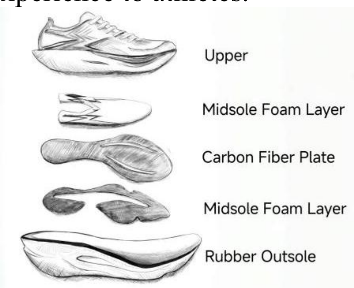

Figure 1. Diagram of the Composition of a Carbon Plate Running Shoe

The advantages of existing research lie in the comprehensive understanding of the basic characteristics and design principles of carbon plate materials, providing a theoretical basis for subsequent innovative designs. However, the limitations of the research are that although existing studies have pointed out the potential of carbon plates in improving sports performance, how to further optimize the design of carbon plates to adapt to the physiological characteristics and sports needs of different athletes is still an unresolved issue. In addition, existing research is also insufficient in terms of energy conversion efficiency and long-term sports performance. Looking internationally, many companies and experts have been able to skillfully use advanced technology such as foot pressure analysis, while in comparison, domestic companies are relatively lacking in the application of these technologies to optimize product design capabilities.

This study aims to address the issue of insufficient optimization in the design of carbon plates in existing research. We plan to use methods such as finite element analysis, biomechanical principle analysis, and energy conversion efficiency evaluation to analyze the carbon plates. Systematic optimization finite element numerical simulation technology [4] has emerged as an effective analysis method in recent years, providing support for human biomechanical analysis and having great application potential in the medical field. Finite element models can biomechanically model and simulate the soft tissues, bones, and midsoles of running shoes [5-6], analyzing the deformation, force, displacement, and stress-strain of various tissue structures under different loads and boundary conditions. Through these methods, we hope to develop carbon plate running shoes that are more in line with ergonomics, more conducive to effective energy conversion, and ultimately improve the sports performance of athletes.

2. Data Collection

Choose an existing non-carbon board running shoe: Depending on the popular running shoe brands on the market, you can choose a running shoe that is suitable for professional long-distance runners.

Now we select a pair of non-carbon-board running shoes on the market as the research object. The data of its running shoes is shown in Table 1:

Table 1. Shoe Data

Shoe length	0.3m
Shoe width	0.12m
Weight	230g(men's)
Midsole thickness	The forefoot is 22mm, and the heel is 31mm
Midsoles	EVA
Backfoot	26mm
Forefoot	18mm
Offset	8mm
Rear foot width	76.5mm
Front foot width	102 mm

3. Data Processing

3.1 Finite Element Analysis of Carbon Plate Geometric Model

Currently, we are analyzing the given non-carbon plate running shoes to determine the design parameters for the carbon plate. Based on the shape and dimensions of the shoe sole, we can establish the shape and size of the carbon plate. We utilize relevant drawing tools to create a three-dimensional model of the carbon plate for further finite element analysis.

Employing the Finite Element Analysis (FEA) method to evaluate and optimize the design of the carbon plate is an efficient and precise process. FEA, also known as the Finite Element Method, is an engineering mechanics research approach similar to solving continuous problems. Finite elements refer to a series of discrete units interconnected by nodes, which represent the actual continuous domain. By assigning selected functional relationships within each element and integrating them into the whole, we obtain an approximate solution across the entire solution domain, enabling simulation and analysis of real physical systems.

The primary advantage of FEA lies in its ability to reflect internal stress and strain conditions within a structure. It enables the analysis of mechanical characteristics within complex objects with intricate structures, shapes, loads, and material mechanical properties. Furthermore, for research requiring repetitive experiments, FEA allows for the modification of any experimental parameters, significantly reducing costs and time.

FEA assesses the performance of the carbon plate by discretizing it into multiple small blocks, treating it as a system composed of numerous small elements(Figure 2).By solving parameters such as displacement, stress, and deformation, we can rapidly and accurately evaluate the carbon plate's performance.

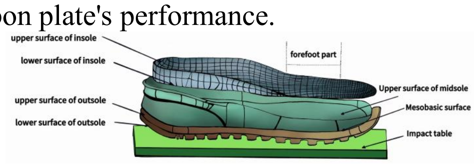

Figure 2. Simulation Analysis Condition Settings

Table 2. Summary of Sports Shoe Simulation Test Data

	heel	Forefoot
Maximum impact force/N	850	790
maximum deformation /mm	6.18	6.36
maximum energy applied/J	7	7
peak G	10.2	9.47

Analysis of the above Table 2 reveals:

Analysis of the above charts and tables reveals:

a) The heel area experiences the highest peak pressure due to not only supporting gravity but also generating force during the push-off phase. The forefoot area follows, primarily absorbing most of the impact force during the landing phase.

b) During long-distance running, the foot undergoes significant rotation from the landing to the support phase, often exceeding normal rotation limits. However, from the support to the push-off phase, rotation is mostly normal. Therefore, the heel area of the shock-absorbing insole should be specifically designed with differentiated inner and outer sides. Considering the natural tendency of foot pronation (outward rotation) during running, the outer side can be designed softer and wider to better accommodate and alleviate the impact force from pronation. The inner side should maintain a certain level of support to stabilize the foot and prevent excessive supination. Additionally, dynamic support elements, such as deformable support structures or elastic materials, can be introduced to automatically adjust support based on the runner's gait and foot rotation.

c) Maximum pressure load rate: Heel > Arch > Forefoot. Therefore, the heel area of long-distance running shoes should prioritize shock absorption design, with the outer side being superior to the inner side. The forefoot area should focus on rebound design.

Given the heel's highest maximum pressure load rate, shock absorption design should be prioritized. High-performance shock-absorbing materials combined with precise structural design maximize shock absorption. Considering the outer side may endure greater impact forces, shock absorption design on the outer side should be superior to the inner side.

The forefoot area should emphasize rebound design to promote energy return and enhance running efficiency. High-elastic materials can be selected, and elastic pads or spring structures can be incorporated into the design to provide additional thrust during push-off. Additionally, the forefoot design should align with the natural curvature of the foot to reduce friction and discomfort.

3.2 Principles of Biomechanics

In running, the midsole of the shoe serves as a crucial shock-absorbing component, and its hardness characteristics directly influence the biomechanical response of the runner's foot. Specifically, there is a notable negative correlation between midsole hardness and shock-absorbing performance: the harder the midsole, the poorer its shock-absorbing effect, resulting in increased force borne by the foot during impact.

During the push-off phase, as midsole hardness increases, the extent of arch height reduction decreases. Changes in the hardness of a rigid midsole running shoe affect the bending stiffness of the sole, with greater hardness leading to increased bending stiffness. Under the same force, increased bending stiffness restricts the movement of the metatarsophalangeal joint during push-off. Multiple studies have indicated that as sole hardness increases, the range of motion of the metatarsophalangeal joint decreases.[7]

When the midsole is softer, it deforms more significantly, leading to a relatively larger distribution of pressure between the outsole and the plantar surface, but with smaller peak pressures. This is likely due to the fact that a softer sole can increase the contact area of the outsole through greater self-deformation and absorb more of the ground reaction force applied to the foot. However, compared to the influence of midsole hardness on outsole pressure, its impact on plantar pressure is relatively minor, as evidenced by a 21% increase in peak outsole pressure and only a 7.5% increase in peak plantar pressure when midsole hardness increases from 10 to 50 Shore A. When the sole is thinner, under high loads, the midsole material is compressed to its deformation limit, becoming harder and losing some of its cushioning effect, resulting in relatively minor changes in plantar pressure with increasing hardness.

Based on biomechanical principles, the hardness characteristics of the running shoe midsole significantly impact the foot's biomechanical response. When designing running shoes, it is essential to consider the balance between midsole hardness and shock-absorbing performance, the relationship between push-off action and arch stability, the distribution and peak pressure changes of sole pressure, as well as the matching of sole thickness and deformation limits, to enhance the comfort and safety of the shoes.[8]

3.3 Factors Influencing the Design of Carbon Plates

3.3.1 The shape and size of the carbon plate

The length and width of the carbon plate have an important influence on the adjustment of the athlete's stride length and sole elasticity. Based on the information searched, the design of the carbon plate can be divided into different categories to suit different types of runners. For example, the carbon plates of the METASPEED EDGE series are designed for stride runners and feature a carbon plate positioned closer to the sole of the foot, with an increased overall width and a wider forefoot, which helps to fully compress the midsole foam when landing, helping the forefoot to gain more propulsion during the start-up phase and thus increase stride length. Instead, the METASPEED EDGE carbon plate is designed for cadence runners, with a wider bend angle and a custom toe cap curvature to help cadence runners maximize control and smoothness during the start-up phase, pushing forward and helping to increase stride length. For long-distance runners, the stride length is longer, and the carbon plate length can be appropriately extended to improve the rigidity and stability of the sole. For example, the full-palm shovel heterogeneous carbon plate used in Li Ning's Feidian 2.0 Elite running shoes reduces the bending of the metatarsophalangeal joints during running, reduces energy loss, and improves the pedaling momentum to ensure the stability and efficiency of the transition from landing to pedaling, which helps to improve stride length and running economy. In general, the wider the carbon plate, the stiffer and more elastic the sole, which allows for better maintenance of the athlete's speed and energy transfer.

3.3.2 The elasticity of the carbon plate

Carbon plate running shoes have a significant impact on running performance due to their unique design. The rigidity and elasticity of the carbon plate can reduce the energy loss when running. For example, the rigidity of the carbon plate helps to stabilize the running shoe and reduce deformation during running, which is especially important for maintaining a correct running posture and reducing energy loss. Especially in long-distance running, stable support can reduce fatigue accumulation. Provides the user with additional propulsion, resulting in increased running efficiency. To determine the minimum value of the length and width of the carbon plate, we first need to calculate the minimum thickness of the carbon plate according to the limit.

Now for the running force analysis, when he runs, most of the weight is concentrated on the forefoot, so we designed the carbon plate to cover only the forefoot area. To simplify the model, we can model a carbon plate as a rectangular plate under cantilever boundary conditions, assuming that the carbon plate is L in length, W in width, and h in thickness.

When the athlete's front foot hits the ground, the carbon plate is subjected to a downward load F, and according to the beam theory, the deflection δ of the carbon plate can be expressed as:

$$\delta = \frac{FL^3}{48EI} \tag{1}$$

Where E is the elastic modulus of the carbon plate material, and I is the moment of inertia of the carbon plate cross-section, it can be approximated as a rectangular plate with a cross-sectional area of $\frac{bh^3}{12}$. The minimum thickness h should meet one of the following constraints:

$$b \leq \frac{FL^3}{48EI\delta_{\max}} \tag{2}$$

$\delta_{\max}$ is the maximum bending deflection of the carbon plate, The length of the area covered by the carbon plate is L,Width is W, The moment of inertia of the carbon plate can be calculated as I, The moment of inertia of a rectangular cross-section, denoted as $I = \frac{bh^3}{12}$,Where b is the width of the rectangle, which is the length of the carbon plate. Therefore, we can calculate the minimum thickness $h_{\min}$ of the carbon plate through the formula

$$delta_{\max} = \frac{559.81t_{\max^2}}{8Et_{h^3}} \tag{3}$$

$$I = \frac{Wh_{\min^3}}{12} \tag{4}$$

$$h_{\min} = \frac{559.81L^3}{48ELdelta_{\max}} \tag{5}$$

3.3.3 The weight of the carbon plate

The weight of the carbon plate can also have a significant impact on the reliability and durability of the sole and the characteristics of the athlete's stride.

To reduce the weight of carbon plates, we can use high-strength, lightweight carbon fiber materials while minimizing waste and unnecessary materials during the design and manufacturing process. When the weight of the carbon plate is lighter, the less burden on the foot, which improves the comfort of wearing for a long time. The traditional belief is that the weight of running shoes is negatively correlated with running economy, that is, for every 100 grams of running shoe weight, the energy consumption will increase by about 1%. However, the advent of superrunning shoes has reduced the impact of shoe weight on runners' energy expenditure, such as the Nike Vaporfly NEXT, which can still improve the economy of running even if it increases the mass of runners by about 50 grams. Therefore, when designing carbon plates,it is necessary to pay attention to maintaining the characteristics of light weight, and to balance the relationship between elasticity, stiffness and weight of carbon plates.To calculate the minimum mass of the carbon plate, we need to first perform the force analysis and bending stiffness calculation of the carbon plate, and then use the constraints to determine the appropriate thickness range, and bring in the mass formula to find the minimum mass.

Since the length, width, and thickness range of the carbon plate are known, we can calculate the volume of the carbon plate based on these parameters:

$$V = L * W * h \tag{6}$$

Where L is the length of the carbon plate, W is the width of the carbon plate, and h is the thickness of the carbon plate.

The mass of the carbon plate can be calculated by the following formula:

$$m = rho * V \tag{7}$$

Where rho is the density of the carbon plate, and V is the volume of the carbon plate.

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Since the Young's modulus ($200e^9 Pa$) of carbon plates is known, the density of carbon plates can be calculated using the following formula:

$$rho = \frac{Eh}{2L^3} \tag{8}$$

Then, by determining the range of carbon plate thickness, matlab can be used to solve the problem of minimizing the weight of carbon plates

3.3.4 The elasticity of the carbon plate

The elastic size of the carbon plate in a carbon plate running shoe is critical for running performance. The elasticity of the carbon plate can provide additional propulsion, helping runners save energy while running and improve running efficiency. Resilient carbon plates bend when subjected to foot pressure, absorb impact, and quickly regain shape when the pressure is gone, providing a continuous rebound effect. This feature can increase ground reaction force during the push and extension phase of the run, helping the runner to move forward more efficiently. For the elasticity coefficient of carbon plate running shoes, we know that the harder the sole material of carbon plate running shoes, the greater the elasticity coefficient, the less energy stored in the running shoes, and the less energy that can be fed back to the athlete, the more energy the athlete consumes.

By consulting the relevant information, we can know that to measure the performance of running shoes, we also need to pay attention to the safety and stability of running shoes, and it is not advisable to blindly improve the elasticity and stubbornness coefficient of running shoes. At the same time, softer shoes will also thicken the size of the shoes, increase the quality of the shoes, and are not conducive to the performance of long-distance running. Overall, from the point of view of the energy consumed by the athlete, the best carbon plate running shoes should be a reasonable combination of hardness and quality of the running shoes.

3.3.5 The thickness of the carbon plate

In order to ensure that the carbon plate can be embedded in the existing non-carbon plate running shoes, it needs to be designed according to the size and shape of the sole. Thicker midsoles are typically equipped with high-performance foam materials such as Pebax or ZoomX, which have excellent flexibility and high resilience to provide good energy return while running. The carbon plate is embedded in such a midsole that stores energy when the runner's foot hits the ground and releases it quickly when it leaves the ground, increasing the forward momentum. The thick-soled design provides better impact absorption and reduces impact on runners' ankles and knees, which is beneficial for preventing sports injuries and improving comfort. At the same time, the thickness and width of the carbon plate also need to be adjusted according to the athlete's foot shape and running habits to ensure foot support and comfort when wearing shoes to achieve the best support and rebound effect.

$$\min F(h) = w_1 * \frac{sigma_{\max}}{S} + w_2 * rho * V + w_3 * \frac{EA}{2h} \tag{9}$$

Now our team based on the above data, when the carbon plate shape and material are determined, we can build an optimization model through the following objective function to find the optimal carbon plate thickness h_{opt} :

$$h_{\min} \le h \le h_{\max} \tag{10}$$

Thereinto, $sigma_{\max}$ is the maximum stress of the carbon plate

S is the cross-sectional area of the carbon plate;

rho is the density of the carbon plate material;

V is the volume of the carbon plate;

E is the modulus of elasticity of the carbon plate material;

A is the cross-sectional area of the carbon plate;

$h_{\min}$ and $h_{\max}$ are the minimum and maximum values of carbon plate thickness

And w_1 , w_2 , and w_3 are the weight coefficients of each objective function term, which we can adjust according to specific situations.

4. Data Solving Process

4.1 Establish Multi-objective Analysis

Suppose the added carbon plate is a rectangle, its length is L , the width is W , the thickness is h , the athlete's weight is M , the cadence is F , the step length is D , the elasticity coefficient of the carbon plate is E , and the elasticity coefficient of the

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buffer layer is G.

Objective function 1: The rebound capacity of the carbon plate

It is determined by the elastic coefficient E of the carbon plate and the shape of the carbon plate.

$$E = \frac{3ML^3}{4WT^3} \tag{11}$$

Objective function 2: The supporting capacity of the carbon plate

The supporting capacity of the carbon plate can be determined by the shape and thickness of the carbon plate

$$S = \frac{Mg}{LW} \tag{12}$$

where is the acceleration due to gravity.

Objective function 3: energy utilization efficiency (conversion rate of gravitational potential energy to elastic potential energy)

The elasticity of the sole material and the elasticity of the sole material will have a significant impact on cushioning as well as backpedaling. In the selection of sole materials, the stiffness and toughness under impact load should be fully considered. Stiffness is able to resist deformation, whereas hardness is able to resist impact loads.The impact load is different for each athlete during the run and can be measured experimentally. If an athlete's maximum kicking force is $3000N$,and the forefoot area of the sole is $100cm^2$,the average impact load is $30N / cm^2$.[9]

Through the relationship between sole deformation and impact force, it is known that

When it reaches its maximum power, its effect is a huge limit of dragging the ground, that is, the limit of protection so that the shock absorption range of the sole can be determined, so the objective function is the maximum deformation of the sole, which is denoted as:

$$f_{\max} = k(1 + 4\frac{Am}{k})^{\frac{1}{2}} * \left[\frac{1}{1 + \frac{4Am}{k}} - 1\right]^{\frac{1}{2}} \tag{13}$$

From the perspective of energy conversion, due to the elasticity, part of the elastic potential energy can be stored to realize the secondary utilization of energy and increase the pushing force when pedaling.

Since the carbon plate embedded in the sole of the shoe mitigates the impact of the ground on the athlete, the deformation of the carbon plate is an important factor affecting the conversion rate, so the objective function is defined as the conversion rate between gravitational potential energy and elastic potential energy according to the elastic potential energy formula.

$$\int_x^0 -kdx = \frac{1}{2}k\Delta x_i^2 \tag{14}$$

The maximum conversion rate of gravitational potential energy to elastic potential energy can be obtained, denoted as:

$$\varphi_i = \frac{Mg\Delta x_i \cos\alpha_i}{Mgx_i} \tag{15}$$

Among them, $\varphi_i(i = 1、2)$ the maximum conversion rate of gravitational potential energy and elastic potential energy of toe cap and heel is described, respectively.

Objective function 4: Add the mass of the carbon plate

The quality of the shoe also has an impact on running performance. Shoes are an add-on to the human body itself, and the greater the mass of the shoe, the greater the cumulative effect of energy expended during the whole running process [10]. Assuming that the distance of running is s and the mass of each running shoe is m, the energy consumed by the human body on each running shoe is about $1.5mgs(J)$ .This indicates that with the increase of distance, the energy accumulation effect will become more and more obvious. Therefore, on the premise of ensuring a certain strength, it will be very effective to appropriately reduce the quality of the shoes.

The formula for calculating the mass of the carbon plate:

$$m = \rho LWT \tag{16}$$

4.2 Model Solving

Establish the objective function:

$$\max E = \frac{3ML^3}{4WT^3} \tag{17}$$

$$\max S = \frac{Mg}{LW} \tag{18}$$

$$\max H = \varphi_i = \frac{Mg\Delta x_i \cos\alpha_i}{Mgx_i} \tag{19}$$

$$\min m = \rho LWT \tag{20}$$

Constraints:

$$\begin{cases} \alpha = \tan^{-1}\left[\frac{(x_1 - x_2)}{w}\right] \\ 10mm < L < 20mm \\ 5mm < W < 20mm \\ \rho = 1.8g / cm^3 \\ M = 55kg \end{cases} \tag{21}$$

To determine the minimum length and width of a carbon plate, the minimum thickness of the carbon plate must first be calculated according to its limits.

Now for the running force analysis, when he runs, most of the weight is concentrated on the forefoot, so we designed the carbon plate to cover only the forefoot area.

To simplify the model, we can model a carbon plate as a rectangular plate under cantilever boundary conditions, assuming that the carbon plate is L in length, W in width, and h in thickness. When the athlete's front foot hits the ground, the carbon plate is subjected to a downward load F . Assuming that F is the weight of the athlete's weight, take 55kg; $\delta_{\max}$ is the maximum bending deflection of the carbon plate, which can be calculated according to the force analysis formula. According to the data, the area covered by the carbon plate is L in length and W in width, and the moment of inertia I of the carbon plate can be calculated, that is, the moment of inertia of the rectangular section, which is $I = \frac{bh^3}{12}$, where b is the width of the rectangle, that is, the length of the carbon plate.

Therefore, we can calculate the minimum thickness $h_{\min}$ of the carbon plate. The minimum thickness limit allows you to solve the problem of the minimum area of the rectangle to achieve a minimum of the length and width of the carbon sheet. [9]

Since the thickness of the carbon plate ranges from 0.001m to 0.02m, the model established above uses the fmincon function in MATLAB to solve the problem of minimizing the mass of the carbon plate, and the optimal solution of the objective function is calculated as Table 3:

Table 3. Optimal Solution of Objective Function

length	0.1672 m
width	0.0119 m
thickness	0.0079 m
quality	0.0134 kg

After analysis, it is not difficult to see that the $[H_1(x, y) - H_2(x, y) + F(x, y)]$ part of the sole has not changed, and the part that has changed is C , and the elastic modulus of the shoe will change after adding the carbon plate, and the new elastic modulus is E .

When the shape of the carbon plate surface is exactly the same as that of the sole, we make the assumption that the elastic modulus of the composite is linearly weighted by the two elastic modulius, i.e., as follows:

$$E' = rE_1 + (1 - r)E_2 \tag{22}$$

E_1 is the elastic modulus of the sole of ordinary running shoes, E_2 is the elastic modulus of the carbon plate, and R reflects the mixed ratio of the carbon plate, that is, the proportion of the carbon plate components. The modulus of elasticity of the soles of ordinary running shoes is usually between $10 - 50MPa$ (megapascals). The modulus of elasticity of carbon plates is usually between $120 - 350MPa$ (megapascals). So

$$\frac{E_1}{E'} = \frac{E_1}{KE_1 + (1-K)E_2} = K \tag{23}$$

$$H_2(x)=\frac{E_1}{E'}H_2(x)=KH_2\left(x\right) \quad (24)$$

In the following section, K is used as the focus of the study to analyze the thinning coefficient $K=\frac{E_1}{KE_1+\left(1-K\right)E_2}$ after adding carbon plates, reflecting the degree of thinning of the sole after adding carbon plates. When E_1 takes $10MPa$ E_2 takes $235GPa$ the thinning coefficient K is in Figure 3

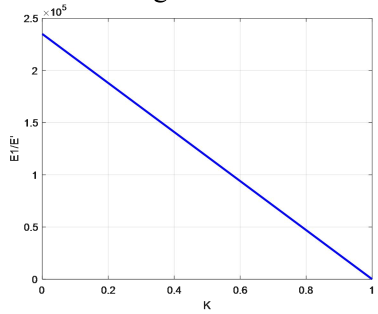

Figure 3. Thinning Factor

The x-axis represents the thinning coefficient K, the y-axis represents the $\frac{E_1}{E'}$ value, and the function image shows an oblique linear trend from left to right, and the E_1 value gradually decreases with the increase of K, and finally approaches 0. It is now possible to use the particle swarm optimization algorithm to optimize the elastic modulus analysis of K, E_1 and E_2 materials to find the combination of parameters that can maximize the $\frac{E_1}{E'}$ value. Here we take $\left[10,50\right]$ and $\left[120e^9,350e^9\right]$, respectively. During the optimization process, the Global Search object was used for global search, and multiple optimization parameters were set to improve the search efficiency and accuracy.

Therefore, within a given range of parameters, the value of $\frac{E_1}{E'}$ is a combination of maximizing parameters as

Thinning coefficient $K:0.06$

Modulus of elasticity of materials $E_1:21$

Modulus of elasticity of materials $E_2:200e^9$

5. Summary

The core objective of this study was to finely tune the key component of high-performance running shoes — the carbon plate — by employing advanced analytical techniques, including but not limited to finite element simulation, biomechanical theory, and multi-objective optimization strategies. The study initially established the geometric model of the carbon plate and analyzed its mechanical properties using the finite element method. Later, based on the principle of biomechanics, the influence of the main factors such as elasticity, size, weight, and thickness of the carbon plate on the performance of running shoes was deepened.

During the data processing phase, the research team selected an existing non-carbon plate running shoe as a benchmark, thoroughly recording and analyzing its performance parameters. Based on this foundation, a three-dimensional digital model of the carbon plate was constructed and an in-depth simulation test was conducted using simulation technology. Through finite element analysis, it was found that the forefoot area experiences the highest peak pressure during the running process, followed by the heel, providing an important basis for subsequent shock absorption and rebound design. Additionally, the study found that athletes have a significant degree of foot rotation during long-distance running, especially from the landing to the support phase. Therefore, the heel part of the shock-absorbing insole needs to be specially customized to adapt to this movement characteristic.

Based on biomechanical principles, a detailed analysis was conducted on how the hardness of the midsole of running shoes affects its cushioning performance. The results showed that the greater the hardness of the midsole, the worse the cushioning performance and the higher the impact force on the foot. At the same time, changes in hardness can also affect the range of motion of the metatarsophalangeal joint, thereby affecting running efficiency.

In the process of optimizing the design of the carbon plate, the research team comprehensively considered multiple key dimensions such as the shape, size, weight, elasticity, and thickness of the carbon plate.

Through multi-objective analysis, the optimal length (0.1672m), width (0.0119m), mass (0.0134kg), and minimum thickness (0.0079m) of the carbon plate were determined. The precise setting of these parameters aims to ensure the rigidity and stability of the carbon plate while achieving lightweight material, thereby greatly enhancing the wearing experience.

The study found that optimizing the length of the carbon plate can effectively enhance the rigidity and stability of the sole, thereby improving the athlete's speed and energy transfer efficiency; adjusting the width of the carbon plate helps to maintain the elasticity and support performance of the sole; using high-strength lightweight carbon fiber materials and implementing minimized thickness design not only achieves lightweight of the carbon plate and reduces the burden on the runner's foot but also ensures optimal balance in the running shoe's ability to absorb impact and provide rebound force through the optimization of the carbon plate's elasticity.

In summary, this study has successfully developed a new type of carbon plate running shoe through a series of systematic optimization strategies. This running shoe has not only made breakthroughs in ergonomic design but also shows significant advantages in promoting effective energy conversion and enhancing athletes' sports performance. The results of this study have not only provided solid scientific support for the innovative design of carbon plate running shoes but also opened up new horizons for the future development of sports equipment.

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