Analysis of the Effect of Nonlocal Factors on the Vibration of Euler-Bernoulli Beams

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Abstract: Currently, the Euler-Bernoulli beam theory relies mainly on the classical continuous medium and local elasticity theories in vibration analysis, but fails to take into account the length interactions between atomic lattices, which makes it difficult to accurately reflect the mechanical properties of beams. Therefore, the goal of this study is to develop a novel method to accurately calculate the mechanical properties of beams. The method is used in combining the Eringen nonlocal theory and extending the Euler-Bernoulli beam theory to construct a nonlocal physical model of beams applicable to arbitrary loads and to verify its degradation. The model is solved by the Nigam-Jennings method, and the effects of nonlocal factors and beam parameters on its self-oscillation frequency and deformation are explored. It is found that the effect of the nonlocal factors on the vibration frequency is negligible after the beam length reaches a certain threshold, while the effect of the nonlocal factors on the vibration frequency and amplitude is significantly enhanced when the vibration mode order increases.

Keywords: Nonlocal Factors; Euler-Bernoulli Beam Theory; Frequency; Amplitude; Nigam-Jennings Method

1. Introduction

With the rapid progress of nanoscience and technology, the unique physical and mechanical properties exhibited by nanoscale materials have become a hotspot of scientific research, and these excellent properties are largely affected by the complex and fine interaction forces between their molecular structures. This phenomenon not only poses a serious challenge to the traditional local theoretical models, but especially stretches the ability to accurately describe the mechanical behavior of materials at the nanoscale [1]. To address this challenge, distinguished scholars such as Eringen and Edelen [2-7], building on the solid foundation of the work of pioneering scholars such as Kroner [8], Rivlin [9], and Krumhansl [10], have conducted in-depth explorations to reveal the nonlocal residual properties of the field, which involve a number of critically important aspects such as the body force, mass, entropy, and internal energy. They have clearly defined these nonlocal residuals and their intrinsic relations by means of strict thermodynamic principles, and then pioneered the theory of nonlocal elasticity. This theory not only systematically elaborates the core principles of continuum field theory, differential continuum coupled field theory, and polar continuum theory [11], but also clearly defines the essential difference between nonlocal elasticity theory and classical local elasticity theory, which provides new perspectives and tools for understanding the mechanical behavior of nanoscale materials [12, 13].

Since its introduction, the nonlocal elasticity theory has demonstrated strong explanatory power and predictive ability in several scientific fields. In the field of nanomaterials, the theory has successfully solved the problem of carbon nanotube buckling, which has long plagued the scientific community [14], and opened up a new way for the application of nonlocal effects in the stability analysis of nanostructures. In addition, in the longitudinal vibration analysis of nonlocal rods, the nonlocal elasticity theory also plays a crucial role, providing a strong theoretical support for the accurate prediction of the vibration behavior of nanorods [15, 16].

On this basis, scholars such as Bao Siyuan [14] further promoted the development of the nonlocal elasticity theory, and they thoroughly investigated the vibration characteristics of nonlocal Euler-Bernoulli beams by innovatively improving the spectral geometry method of Fourier series. This method effectively overcomes the limitations of the traditional Fourier series method in dealing with the discontinuities of the boundary conditions and the convergence speed, and provides a new way to accurately analyze the vibration characteristics of nonlocal Euler-Bernoulli beams. This research result not only deepens our understanding of the influence of nonlocal effects in the vibration behavior of beam structures, but also provides more solid theoretical support and practical guidance for the optimal design, performance evaluation, and disaster prevention and mitigation of nano beam structures.

However, although significant progress has been made in the nonlocal Euler-Bernoulli beam theory, it still faces some urgent problems. Specifically, the current study has not yet fully revealed the intrinsic connection between the nonlocal factors and the beam vibration characteristics and beam structural parameters under arbitrary loading conditions. In addition, the existing solution process is relatively complicated, which is not conducive to its wide application in practical engineering. Therefore, this study is devoted to the development of a more efficient and accurate method to calculate the mechanical properties of beams. The method closely combines Eringen's nonlocal theory with Euler-Bernoulli beam theory to construct a nonlocal physical model of beams applicable to arbitrary loading conditions. Through this model, the specific influence mechanisms of the nonlocal factors as well as the structural parameters of the beams on the beams' self-resonance frequency and deformation behavior will be analyzed in depth, aiming to provide a more solid theoretical support and practical guidance for the optimal design, performance evaluation, and disaster prevention and mitigation of beam structures.

2. Computational Model

An arbitrary intercept of a beam dx microelement segment is shown in Figure 1. The forces acting on this dx microelement segment of the beam in Figure 1 are bending moments on the left and right cross-sections, respectively, $M(x,\tau,t), \frac{\partial}{\partial x}M(x,\tau,t)dx$, and shear forces on the left and right cross-sections, respectively, $Q(x,\tau,t), \frac{\partial}{\partial x}Q(x,\tau,t)dx$. An arbitrary accidental load $F(t)$, the inertial force in the inertial force of the beam of the dx microelement segment, $f(x,t) = \rho A\frac{\partial^2 w(x,t)}{\partial t^2}$. In Figure 1 τ is a nonlocal parameter, $\tau = \frac{a \cdot e_0}{L}$, e_0 are constants corresponding to each material, a is an internal characteristic length (e.g., lattice parameter, particle distance), and L is an external characteristic length (crack length, wavelength).

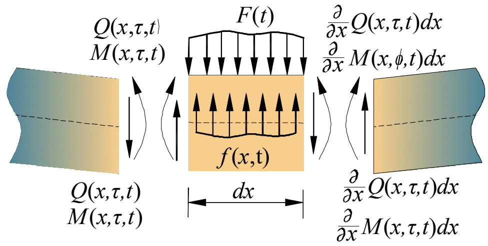


Figure 1. Microelementary Segment of Beam under Arbitrary Loading

(1) Equilibrium equations

$$\frac{\partial^2}{\partial x^2}M(x,\tau,t) =- F(t) + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} \quad (1)$$

(2) Eigenstructural equations for nonlocal elastic problems

$$\sigma_{ij,j}^L = \rho_s[1-\tau^2\nabla^2]\ddot{u}_i \quad (2)$$

Where ρ_s is the density of the beam structure material (Kg/m^3),τ is a non-local parameter that responds to local features.

According to the law of conservation of mass, the shear and internal forces of a beam have the following relationship:

$$M(x,\phi,t) =- \int_A y\,\sigma_x(x,\phi,t)dA \quad (3)$$

From Eqs. (1), (2) and (3):

$$-EI\frac{\partial^4}{\partial x^4}w(x,t) =- F(x,t) + \rho A\frac{\partial^2}{\partial t^2}w(x,t) - \phi^2\left[-\frac{\partial^2}{\partial x^2}F(x,t) + \rho A\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial t^2}w(x,t)\right] \quad (4)$$

3 Model Solving and Analysis

The solution of Eq. (4) is of the form:

$$w(x,t) = q(t)A_n \sin\frac{n\pi x}{L} \quad n = 1,2,\cdots\infty \quad (5)$$

Where $A_n \sin\frac{n\pi x}{L}$ represents the shape of the vibration, which does not vary with time; $q(t)$ indicates the amplitude over time; A_n is a constant, determined by the initial

conditions, and in this paper $A_n = 1$.
Substituting Eq. (5) into Eq. (4), then:

$$a_0 \frac{\partial^2}{\partial t^2} q(t) + b_0 q(t) = 0 \quad (6)$$

Where $a_0 = \frac{\rho A}{EI}\left[1 + \tau^2\left(\frac{n\pi}{L}\right)^2\right], b_0 = -\left(\frac{n\pi}{L}\right)^4$

Eq. (6) is a chi-square differential equation with a generalized solution: $q(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$,

Where $\omega_n = (n\pi)^2 \sqrt{\frac{EI}{\rho A[L^2+(\tau n\pi)^2]L^2}}$, if we take the nonlocal parameter τ=0 ,then $\omega_n = (n\pi)^2 \sqrt{\frac{EI}{\rho A L^4}}$, degenerates to the intrinsic frequency of the classical Euler-Bernoulli beam.

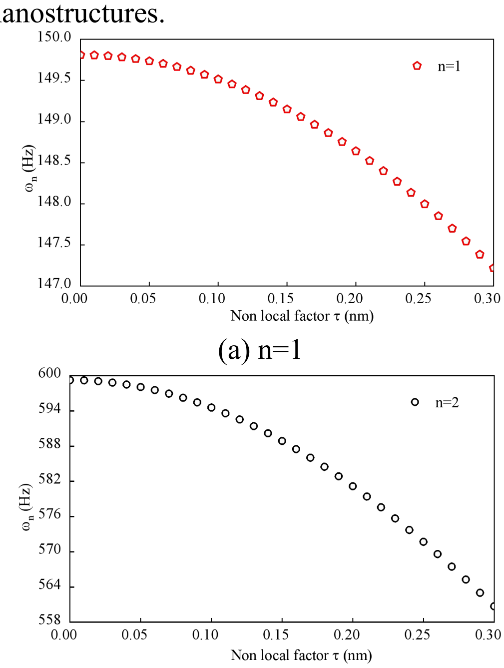
Take the beam width $a = 0.4m$, height $h = 0.6$ and modulus $E = 1 \times 10^{10} N/m^2$. In order to analyze in depth the specific effect of the non-local factors on the beam self-resonance frequency, observe parts (a) and (b) in Figure 2. The following conclusions are drawn from the comparison:

With the beam length kept constant, this study systematically investigates the effect of non-local factors on the self-oscillation frequency of the beam. The results show that the self-oscillation frequency of the beam exhibits a definite decreasing trend with the gradual increase of the non-localization factor. This phenomenon clearly indicates that at the nanoscale, the nonlocal effect has a significant modulation effect on the vibration characteristics of the beam, and this modulation effect becomes more and more significant with the increase of the nonlocal factor.

Further analysis of the data reveals that the absolute value of the rate of change of the beam vibration frequency (i.e., the rate of frequency reduction) decreasing with the increase of the non-localization factor also shows a gradually increasing trend. This means that the rate of decrease of the beam self-oscillation frequency accelerates with the continuous increase of the non-localization factor, indicating that the influence of the non-localization effect on the beam vibration characteristics shows an accelerated and enhanced trend in the dynamic change.

In order to understand this phenomenon more deeply, this study also compares and analyzes the vibration behaviors of beams with different nonlocal factors, and finds that the increase of nonlocal factors not only leads to the decrease of the beam self-oscillation frequency, but also significantly affects the vibration modes and response characteristics of beams. These findings provide important clues for a comprehensive understanding of the dynamic change laws of nonlocal effects in the vibration behavior of beams, and also provide a solid theoretical foundation for the optimal design, performance evaluation, and dynamic response prediction of nanoscale beam structures.

In summary, through a systematic comparative analysis, this study not only reveals the significant effect of nonlocal factors on the beam self-oscillation frequency, but also probes deeply into the tendency of this effect to intensify with the increase of the nonlocal factors and the physical mechanism behind it. These findings are of great significance for an in-depth understanding of the vibrational properties of nanoscale beams and their behavioral performance in practical engineering applications, and provide important theoretical guidance and practical basis for the design and optimization of nanostructures.



(b) n=2

Figure 2. Relationship between Non-Local Factors and Beam Self-Oscillation Frequency

The Nigam-Jennings method is applied to solve the beam nonlocal physical model. Assuming that in the time period $[t_i, t_{i+1}]$, then the following relation exists for the beam amplitude, then:

$$a\frac{\partial^2}{\partial t^2}q(\tau_1)+bq(\tau_1)=F_i+\alpha\tau_1 \quad (7)$$

Take the initial condition: $q(\tau_1)|_{\tau_1=0}=q_i, \dot{q}(\tau_1)|_{\tau_1=0}=\dot{q}_i$

$$q_F(\tau_1)=\frac{1}{k}(F_i+\alpha_i\tau_1) \quad (8)$$

Substituting the full solution into the initial conditions, then:

$$\begin{cases} q(\tau_1)=\frac{p_i}{k}+\frac{\alpha_i}{k}\tau_1+\left(q_i-\frac{p_i}{k}\right)cos\,(\omega_n\tau_1)+\frac{1}{\omega_n}\left(\dot{q}_i-\frac{\alpha_i}{k}\right)sin\,(\omega_n\tau_1) \\ \dot{q}(\tau_1)=\frac{\alpha_i}{k}-\left(q_i-\frac{p_i}{k}\right)\omega_n\,sin\,(\omega_n\tau_1)+\left(\dot{q}_i-\frac{\alpha_i}{k}\right)cos\,(\omega_n\tau_1) \end{cases} \quad (9)$$

take $\tau_1=\Delta t$,then:

$$\begin{cases} q_{i+1}=A_1q_i+A_2\dot{q}_i+A_3p_i+A_4p_{i+1} \\ \dot{q}_{i+1}=B_1q_i+B_2\dot{q}_i+B_3p_i+B_4p_{i+1} \end{cases} \quad (10)$$

Where

$$A_1=cos\,\omega_n\,\Delta t, A_2=\frac{1}{\omega_n}sin\,\omega_n\,\Delta t, A_3=\frac{1}{k}\left[\frac{1}{\omega_n\Delta t}sin\,\omega_n\,\Delta t-cos\,\omega_n\,\Delta t\right], A_4=\frac{1}{k}\left[1-\frac{1}{\omega_n\Delta t}sin\,\omega_n\,\Delta t\right]$$

$$B_1=-\omega_n\,sin\,\omega_n\,\Delta t, B_2=cos\,\omega_n\,\Delta t, B_3=\frac{1}{k}\left[-\frac{1}{\Delta t}+\omega_n\,sin\,\omega_n\,\Delta t+\frac{1}{\Delta t}cos\,\omega_n\,\Delta t\right], B_4=\frac{1}{k\Delta t}[1-cos\,\omega_n\,\Delta t]$$

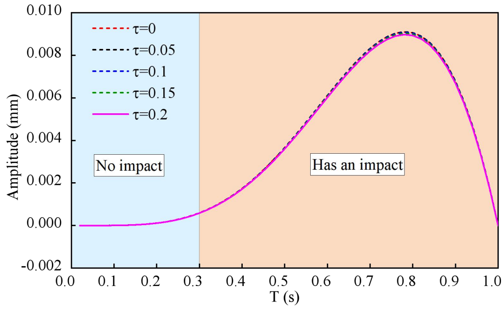
Eq. (10) is the Nigam-Jennings method step-by-step integration formula, according to the amplitude of the moment t_i and the external load to calculate the amplitude of the moment t_{i+1}, given the initial conditions, and constantly according to the above formula cycle, you can get based on the Euler-Bernoulli beam theory in any load beam amplitude.

In order to comprehensively and deeply investigate the specific effects of the nonlocal factors on the beam amplitude, sophisticated computational analyses have been performed in this study and the quantitative relationship between the nonlocal factors and the beam amplitude has been successfully established, as shown in Figure 3. By meticulously comparing parts (a) and (b) in Figure 3, several key and interesting phenomena are observed, and these findings are of great revelation for understanding the vibrational behavior of nanoscale beams.

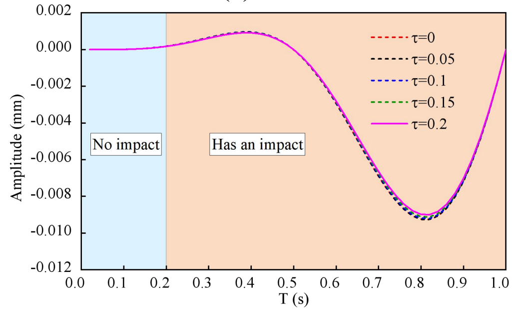
First, it is worth noting that the modulation effect of the nonlocal factors on the beam amplitude becomes more and more significant as the order of the vibrational mode, n, continues to rise. At higher order vibration modes, the nonlocal effect acts as a key regulator, profoundly influencing the vibration behavior of the beam and exerting a more prominent effect on the amplitude. This finding not only further enriches our knowledge of the mechanism of nonlocal effects in nanoscale beam vibration, but also provides a strong theoretical support for the prediction of the dynamic response of beams in higher-order vibration modes.

Secondly, it is also found that in the initial stage of vibration, the effect of the nonlocal factors (or the nonlocal inertial forces generated by them) on the beam amplitude is relatively small, like a fine stream at the beginning. However, with the passage of time, this effect gradually accumulates, just like a trickle of tiny streams converging into a river, eventually leading to significant changes in the amplitude. This phenomenon profoundly reveals the nature of the non-local effect, that is, it is actually a reflection of the complex interactions between the internal plasmas of the material. During the vibration process, such interactions are like accumulating energy, which ultimately affects the amplitude of the beam significantly on long time scales. Therefore, the influence of nonlocal factors on beam amplitude at the nanoscale should never be underestimated, especially in the long-time vibration process, and its modulation is particularly prominent.

In addition, the relationship between the nonlocal factors and the beam amplitude with the vibration order and the vibration time is deeply explored through systematic computational analysis in this study. These findings not only provide a new perspective to deeply understand the vibrational behavior of nanoscale beams, but also provide valuable theoretical guidance for the design and optimization of nanostructures. In the future, we will continue to deepen the research in this field through more experimental validation and theoretical innovation, with the aim of making more breakthroughs in the vibration control and performance optimization of nanoscale beams, and contributing more to the development of nanotechnology.



(a) n=1



(b) n=2

Figure 3. Effect of Non-Local Factors on Beam Amplitude

4. Conclusion

The Eringen nonlocal theory is introduced and combined with the Euler-Bernoulli beam theory to establish a nonlocal physical model of beams under arbitrary loads, and the degradation validation of the model is given. The Nigam-Jennings method is applied to solve the beam nonlocal physical model, and the effects of nonlocal factors and nano parameters on the beam self-oscillation frequency and deformation are analyzed by the self-programmed program, and the following main conclusions are drawn:

(1) The beam vibration frequency decreases when the nonlocal factor increases; the absolute value of the rate of change of the beam vibration frequency with the increase of the nonlocal factor gradually increases.

(2) When the vibration type n increases, the influence of the non-local factor on the beam amplitude is greater; the influence of the non-local factor on the beam amplitude is small in the initial period of time, and the influence of the non-local factor on the beam amplitude gradually increases with the increase of time.

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