Decision Making on the Air to Beyond Visual Range Target Strike of Shipborne Weapons with Adjustable Muzzle Velocity

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Abstract: The biggest difference between shipborne weapons with adjustable muzzle velocity and traditional naval guns is that the muzzle velocity of the projectile can be variable. Based on those characteristics, this paper establishes a hit probability model for air targets. The influence of target encounter points on the single-shot hit probability is analyzed. The changes of the cumulative hit probability of whole-route continuous-shot under the different conditions of fixed muzzle velocity and variable muzzle velocity are studied. Based on this analysis, the strike strategy for over-the-horizon air targets is proposed, that is, long-distance point fire and close-range continuous fire, so as to achieve better strike effect with a lower strike cost.

Keywords: Adjustable Muzzle Velocity; Air Targets; Hit Probability; Strike Strategy

1. Introduction

In comparison to conventional naval guns, shipborne weaponry endowed with adjustable muzzle velocity boasts a multitude of advantages, encompassing swift responsiveness, exceptional interception precision, formidable destructive capability, extended range, and robust comprehensive damage potential [1,2]. Moreover, these advanced weapons are characterized by their capacity to vary both projectile muzzle velocity and firing rate [3]. As nations worldwide intensify their research endeavors into shipborne weapon technologies, such weaponry has progressively emerged as a highly auspicious novel concept within the realm of kinetic weaponry [4], and has commenced its application across diverse combat scenarios, including anti-ship strikes and shore bombardment support missions.

In light of the adjustable muzzle velocity inherent in shipborne weapon projectiles, conventional fire control methodologies for naval guns have become inadequate to satisfy operational demands. Xie and Wu, along with their colleagues, delved into a method aimed at enhancing the hit accuracy and fire density of firepower strikes. This involved fine-tuning the projectile launch velocity to facilitate the simultaneous impact of multiple rounds [5,6]. Xu et al. conducted an analysis of the performance prerequisites for shipborne weapons in modulating projectile muzzle velocity across varying firing ranges, as well as the trends in the weapon's performance indicators at different muzzle velocities. Their findings revealed that, for close-range targets, an elevated muzzle velocity corresponds to an improved single-shot hit probability; however, the advantage of muzzle velocity diminishes as the range increases [7]. Wu et al. employed the Cramer-Rao lower bound theory to formulate a hit probability model for projectiles targeting maritime objectives. They further scrutinized the influences of firing errors, target distance, projectile elevation angle, and muzzle velocity on the target hit probability [8,9]. Zhang et al. developed hit probability models for both single-shot and continuous full-trajectory firing of projectiles against aerial targets. They systematically examined the impacts of target distance and projectile muzzle velocity on hit probability under both firing conditions, and proposed a balanced full-trajectory firing strategy. This strategy seeks to maximize single-shot hit probability within the effective range and achieve the simultaneous impact of multiple rounds at the maximum firing range [10]. Wang et al. established a mathematical model for the simultaneous impact of multiple electromagnetic gun rounds, completing an error analysis and probabilistic error calculation for this phenomenon [11]. Pei et al. undertook a comprehensive study on the dynamic behavior of integrated electromagnetic launch projectiles, utilizing a combination of COMSOL simulation, theoretical analysis, and numerical methods

[12]. Yan et al. investigated the dynamic characteristics of electromagnetic railguns based on COMSOL moving mesh technology, a pursuit of significant importance for the reliability design of electromagnetic guns [13]. At present, the research endeavors concerning attack strategies for shipborne weaponry with adjustable muzzle velocity are predominantly centered around low-velocity maritime targets and aerial targets within visual range. There exists a notable dearth of strategic investigations into continuous firing along the entire trajectory against beyond-visual-range (BVR) aerial targets. In this scholarly work, through the establishment of a hit probability model for projectiles targeting aerial entities, we conduct an in-depth analysis of the influence exerted by the target encounter point on the single-shot hit probability. Additionally, we scrutinize the fluctuations in the cumulative hit probability along the target's entire trajectory under both fixed and variable muzzle velocity scenarios. For BVR aerial targets, we put forth a firing strategy that combines long-range burst firing with short-range continuous firing, aiming to attain superior attack efficacy while minimizing attack costs.

2. The Calculation Model for Firing at Airborne Targets

2.1 The Hit Area of Airborne Targets

The primary mission of anti-air firing operations is to neutralize incoming aerial threats or coerce them into abandoning their attack intentions, thereby safeguarding the security of one's own vessel or the vessel under surveillance. The principal aerial targets encompass enemy bombers, attack aircraft, and an array of anti-ship missiles.

Typically, the examination of the hit area against aerial targets is conducted within the framework of the x-axis and z-axis coordinate systems. The projection area of an aerial target onto a plane perpendicular to the relative velocity of the projectile is denominated as the hit area of the aerial target, symbolized as AT. A target coordinate system is formulated, as depicted in Figure 1, with the origin situated at the target's center of gravity. The x-axis represents the target's longitudinal axis, oriented forward; the z-axis signifies the target's lateral axis, directed to the right; and the y-axis denotes the target's lift axis, perpendicular to the xoz plane and pointing upward.

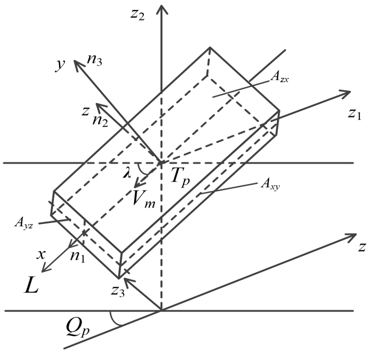

Figure 1. The Projection Relationship of the Three-View Drawing of the Target

2.2 The Hit Probability Model of Airborne Targets

The single-shot hit probability of shipborne weapons against airborne targets is [14]

$$P=\iint_{A_T}\varphi\left(x_1,x_2\right)dx_1dx_2 \tag{1}$$

In the formula, $\varphi(x_1,x_2)$ is the distribution density function of the firing error, and it satisfies

$$\varphi\left(x_1,x_2\right)=\frac{1}{2\pi\sqrt{\left|K_\varphi\right|}}\exp\left[-\frac{1}{2}\left(X-M\right)^T K_\varphi^{-1}\left(X-M\right)\right] \tag{2}$$

Among them, K_φ is the covariance matrix of the firing error, and it is given by

$$K_\varphi=\begin{bmatrix}K_{\varphi11} & K_{\varphi12}\\ K_{\varphi21} & K_{\varphi22}\end{bmatrix} \tag{3}$$

M is the column array of the mathematical expectation of the firing error, and it is given by

$$M=\begin{pmatrix}m_1 & m_2\end{pmatrix}^T \tag{4}$$

When firing at airborne targets, since the selection of coordinate axes is usually inconsistent with the main axes of the dispersion error, there is a coupling relationship between the components of the firing error on the x_1 and x_2 axes. By using the coordinate transformation method, making the directions of the new coordinate axes consistent with the directions of the main axes of the firing error can change the correlation of the error components and decouple the errors from each other.

From this, we obtain the orthogonal matrix K'_φ of K_φ that is,

$$K'_\varphi=\begin{bmatrix}K'_{\varphi11} & 0\\ 0 & K'_{\varphi22}\end{bmatrix} \tag{5}$$

The projections m'$_1$ and m'$_2$ of M on the main axes x'_1 and x'_2 of the comprehensive distribution of the firing error, that is,

$$\begin{cases} m_1' = m_1 \cos\theta_0 + m_2 \sin\theta_0 \\ m_2' = -m_1 \sin\theta_0 + m_2 \cos\theta_0 \end{cases} \tag{6}$$

To calculate the single - shot hit probability P, we have:

$$P = \frac{1}{4}\left[\Phi(c) - \Phi(d)\right]\left[\Phi(e) - \Phi(f)\right] \tag{7}$$

The Laplace function Φ(x) can be replaced by an approximate Laplace function. The integration intervals c, d, e, and f are:

$$\begin{cases} c = \dfrac{m_1' + l}{\sqrt{2K'_{\varphi 11}}}, d = \dfrac{m_1' - l}{\sqrt{2K'_{\varphi 11}}} \\ e = \dfrac{m_2' + l}{\sqrt{2K'_{\varphi 22}}}, f = \dfrac{m_2' - l}{\sqrt{2K'_{\varphi 22}}} \end{cases} \tag{8}$$

In the formula, l is half of the side length of the hit area of the square target, that is, $l = \sqrt{A_T}/2$.

The cumulative hit probability of continuously firing n projectiles at the target is:

$$P_n = 1 - \prod_{i=1}^{n}(1 - P_i) \tag{9}$$

P_i refers to the single-shot hit probability of the i-th firing.

3. Analysis of Firing Decision-Making for Airborne Targets Based on Integrated Kinetic Energy Projectiles

For shipborne weapons with adjustable muzzle velocities, the selection of diverse projectile muzzle velocities tailored to specific attack missions is paramount for augmenting the hit probability. A meticulous analysis of how varying projectile muzzle velocities impact the hit probability serves as a valuable tool in devising attack strategies for shipborne weapons targeting aerial adversaries. Zhang Zhiyong and his associates have undertaken exhaustive examinations of attack decision-making frameworks for within-visual-range aerial targets and put forth a balanced firing strategy spanning the entire trajectory. This scholarly work primarily concentrates on beyond-visual-range (BVR) aerial targets and proposes a corresponding attack strategy.

3.1 Calculation Conditions

It is assumed that the three-view areas of a certain enemy aircraft are as follows: the front-view area A_{yz} = 3.0m^2, the side-view area A_{xy} = 14.5m^2, the top-view area A_{zx} = 23.1m^2. The flight altitude is 500 m, the target pass distance is 100 m, the dive angle is zero, and the target flight speed is: V_{xm} = -300m/s, V_{ym} = 0m/s, V_{hm} = 0m/s. The position of our ship's weapon in the geodetic coordinate system is X = 0m, Y = 0m, H = 10m.

3.2 The Influence of the Distance of the Projectile-Target Encounter Point on the Single-Shot Hit Probability

Assuming the target is on a collision course with our vessel, to gauge the influence of the projectile-target encounter distance-the precise point at which the projectile converges with the target-on the firing efficacy of the shipborne weapon, the maximum value of X$_m$ is set to 18km. Specifically, in the geodetic coordinate system, the position is X$_m$≤18000m, Y$_m$ = 100m, H$_m$ = 500m. Calculate the hit probability P of the incoming airborne target when the muzzle velocities of the projectiles are 1500 m/s and 2000 m/s. Figure 2 shows the relationship curves of the single-shot hit probability with the distance of the projectile-target encounter point when the muzzle velocities of the projectiles are 1500 m/s and 2000 m/s.

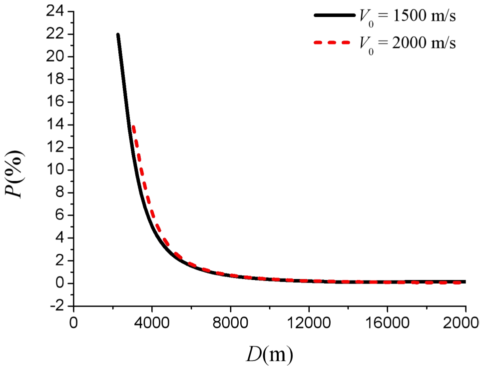

Figure 2. Curve of the Hit Probability Varying with the Distance of the Projectile-Target Encounter Point in Single-Shot Firing

As illustrated in Figure 2, irrespective of the projectile's muzzle velocity, the single-shot hit probability diminishes swiftly as the distance between the projectile and the target at the point of encounter increases. When the target is situated at a distance of less than 14,760 meters, the hit probability, denoted as $P(v_0\text{=2000m/s}) > P(v_0\text{=1500m/s})$. This observation suggests that a higher muzzle velocity is associated with a greater single-shot hit probability, albeit the disparity between the two is relatively minor. As the projectile-target encounter distance surpasses 8,000 meters, the single-shot hit probability plummets to an exceedingly low level and remains largely unaffected by further variations in the encounter distance. This implies that an integrated kinetic energy projectile is rendered ineffective in engaging the target at such extended ranges.

3.3 Analysis of the Hit Probability of the Target Throughout the Entire Flight Path under Fixed Muzzle Velocity and Variable Muzzle Velocity

3.3.1 The Hit Probability of the Target Throughout the Entire Flight Path under a Fixed Muzzle Velocity

In light of the abysmally low single-shot hit probability when the target distance exceeds 8,000 meters, this scholarly work predominantly concentrates on analyzing target distances within this threshold. Assuming a target distance of 8,000 meters, a flight altitude of 500 meters, a target closest point of approach of 100 meters, a dive angle of zero degrees, a flight speed of 300 m/s, and a minimum firing range of 1,000 meters, the cumulative hit probability along the entire trajectory is computed. Given that continuous firing along the entire trajectory is entailed, the charging preparation time for each projectile must be factored into the calculations. Consequently, the determination of the target's future position also incorporates the charging preparation time. The resultant findings are presented in Table 1.

Table 1. The Hit Probability of the Target Throughout the Entire Flight Path when the Muzzle Velocity V_0=1200m/s

Serial Number	Single-shot Hit Probability P(%)	Cumulative Hit Probability Pn(%)	Projectile Flight Time T(s)	Hit Distance D(m)	Current Distance D(m)
1	0.212	0.212	5.784	5634.831	8000
2	0.3	0.51	5.192	5163.418	7353.434
3	0.43	0.939	4.618	4687.743	6707.144
4	0.63	1.563	4.063	4207.823	6061.219
5	0.934	2.482	3.545	3741.009	5415.791
6	1.457	3.903	3.021	3248.582	4771.059
7	2.387	6.197	2.519	2753.186	4127.352
8	4.179	10.118	2.04	2255.414	3485.235
9	7.987	17.297	1.587	1757.235	2845.788
10	16.606	31.03	1.165	1265.185	2211.325

Employing the same methodological approach, the cumulative hit probabilities along the entire trajectory for muzzle velocities spanning from V_0=1200m/s to V_0=2000m/s are compiled. The cumulative hit probabilities along the entire trajectory for fixed muzzle velocities are tabulated in Table 2.

Table 2. Comparison of the Hit Probability of the Target Throughout the Entire Flight Path under Fixed Muzzle Velocities

Fixed muzzle velocity V_0(m/s)	Number of Projectiles Fired (rounds)	Cumulative hit probability Pn(%)
1200	10	31.03
1300	9	34.85
1400	7	15.45
1500	6	11.48
1600	5	7.02
1700	5	10.81
1800	4	4.78
1900	4	6.80
2000	3	2.14

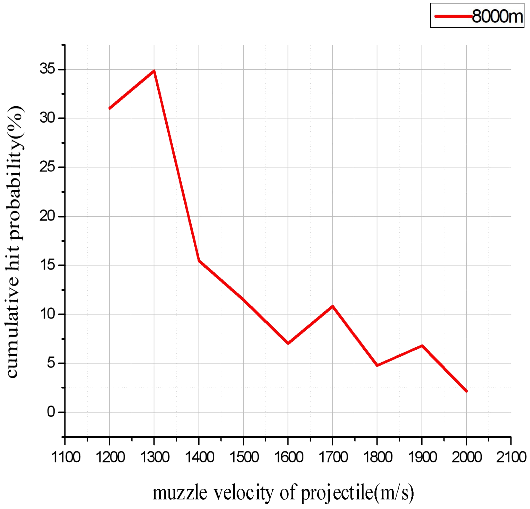

Figure 3. Curve of the Cumulative Hit Probability Throughout the Entire Flight Path Varying with the Muzzle Velocity of the Projectile When the Target Distance Is 8000 Meters

When engaging a target positioned at a

distance of 8,000 meters with varying fixed muzzle velocities, a comprehensive analysis of the resultant data is undertaken. As clearly demonstrated in Table 1, the hit probability associated with the final shot-where the target is successfully struck at the minimum firing range-is notably higher than the single-shot hit probabilities recorded prior. This observation underscores the fact that the hit probability escalates as the target draws nearer. From the data presented in Table 2, it is discernible that as the projectile's muzzle velocity increases, the number of projectiles discharged throughout the entire trajectory diminishes, consequently leading to a proportional decline in the cumulative hit probability. However, it is noteworthy that for an equivalent number of projectiles fired, a firing strategy employing a higher muzzle velocity yields a superior cumulative hit probability. As illustrated in Figure 3, the cumulative hit probability along the entire trajectory attains its zenith when the muzzle velocity is within the range of 1,200 to 1,300 m/s. Nonetheless, this approach entails a considerable expenditure of ammunition. As the fixed muzzle velocity is augmented, the number of projectiles consumed throughout the entire trajectory decreases, yet the cumulative hit probability plummets significantly, thereby rendering it arduous to attain the desired engagement outcome. In summation, for engagement plans against beyond-visual-range (BVR) aerial targets utilizing fixed muzzle velocities, a lower muzzle velocity-spanning from 1,200 to 1,300 m/s-ought to be favored to guarantee a reduced minimum firing range and augment the hit probability of the engagement.

3.3.2 The Hit Probability of the Target Throughout the Entire Flight Path under Variable Muzzle Velocity

The calculation conditions under variable muzzle velocity are the same as those under fixed muzzle velocity.

Table 3. Statistics of the Hit Probability of the Target Throughout the Entire Flight Path When the Target Distance Is 8000 Meters

Serial Number	The muzzle velocity of the projectile V_0(m/s)	Single-shot hit probability P(%)	Cumulative hit probability P_n(%)	Projectile flight time T(s)	Hit distance D(m)
1	1397	0.218	0.218	4.855	5674.838
2	1221	0.34	0.557	4.894	4998.474
3	1189	0.498	1.053	4.443	4499.697
4	1173	0.733	1.778	3.961	4027.465
5	1169	1.079	2.838	3.477	3584.774
6	1160	1.669	4.46	3.004	3120.473
7	1152	2.685	7.025	2.55	2661.459
8	1150	4.549	11.255	2.101	2207.065
9	1150	8.291	18.613	1.669	1753.878
10	1150	16.304	31.882	1.264	1305.369

Suppose the target distances are 7000 meters and 6000 meters, and the other flight path conditions are the same. The statistics of the hit probability throughout the entire flight path are as follows.

Table 4. Statistics of the Hit Probability of the Target Throughout the Entire Flight Path under the Condition of a Target Distance of 7000 Meters

Serial Number	The muzzle velocity of the projectile V_0(m/s)	Single-shot hit probability P(%)	Cumulative hit probability P_n(%)	Projectile flight time T(s)	Hit distance D(m)
1	1212	0.364	0.364	4.824	4905.761
2	1185	0.533	0.896	4.361	4414.874
3	1171	0.786	1.674	3.876	3945.712
4	1168	1.161	2.816	3.391	3503.438
5	1158	1.809	4.574	2.925	3039.649
6	1151	2.931	7.371	2.472	2581.891
7	1150	5.021	12.023	2.024	2128.049
8	1150	9.284	20.19	1.596	1675.153
9	1150	18.408	34.882	1.196	1228.595

Table 5. Statistics of the Hit Probability of the Target Throughout the Entire Flight Path When the Target Distance Is 6000 Meters

Serial Number	The muzzle velocity of the projectile V_0(m/s)	Single-shot hit probability P(%)	Cumulative hit probability P_n(%)	Projectile flight time T(s)	Hit distance D(m)
1	1177	0.656	0.656	4.099	4161.092
2	1164	0.976	1.625	3.622	3697.179
3	1162	1.462	3.063	3.144	3257.242
4	1154	2.321	5.313	2.684	2797.162
5	1150	3.864	8.972	2.234	2341.66
6	1150	6.878	15.233	1.794	1888.276
7	1150	13.264	26.477	1.381	1437.443
8	1150	26.309	45.82	0.998	1001.027

Scrutinizing Tables 3 through 5, it becomes apparent that within the variable muzzle velocity attack paradigm, the projectile's muzzle velocity gradually wanes as the distance between the projectile and the target narrows, while the single-shot hit probability concurrently ascends. As the target distance contracts from 8,000 meters to 6,000 meters, the cumulative hit probability along the entirety of the trajectory experiences an enhancement, and the quantity of projectiles expended also dwindles. This suggests that initiating the assault when the target is in closer proximity results in a reduced expenditure of ammunition and attains a more efficacious engagement outcome.

4. Conclusion

Drawing upon the distinctive attributes of shipborne weapons endowed with adjustable muzzle velocity, a hit probability model tailored for shipborne weapons engaging aerial targets has been meticulously constructed. For beyond-visual-range (BVR) aerial targets, an in-depth analysis has been conducted on the impact of the projectile-target encounter point on the single-shot hit probability, as well as the fluctuations in the cumulative hit probability along the target's entire trajectory under both fixed and variable muzzle velocity scenarios. The findings unveil the following insights:

(1) When aerial targets are situated beyond the 8,000-meter threshold, the single-shot hit probability of employing an integrated kinetic energy projectile plummet to an insignificantly low level, rendering it futile for engaging the target effectively.

(2) In the context of continuous firing along the entire trajectory for targets within 8,000 meters, when the muzzle velocity is held constant, the cumulative hit probability reaches its zenith when the muzzle velocity is within the range of 1,200 to 1,300 m/s. However, this approach demands a substantial outlay of ammunition, thereby escalating the engagement costs. As the fixed muzzle velocity escalates, the number of projectiles consumed along the entire trajectory diminishes, yet the cumulative hit probability experiences a significant decline, culminating in a less effective engagement outcome.

(3) In the scenario of continuous firing along the entire trajectory for targets within 8,000 meters, when the muzzle velocity is variable, the cumulative hit probability along the entire trajectory escalates as the target distance decreases, and the number of projectiles utilized also declines. This implies that initiating the attack when the target is in closer proximity results in a reduced expenditure of ammunition and achieves a more efficacious engagement outcome.

Grounded on the aforementioned analysis, an attack strategy for shipborne weapons with adjustable muzzle velocity against BVR aerial targets is proposed:

(1) In instances where the target is situated at a substantial distance, exceeding 8,000 meters, it is advisable to adopt a burst-fire mode. This approach serves to perturb the target's regular flight pattern and hinder its combat operations.;

(2) When the target is within a closer proximity, specifically within the 8,000-meter range, a strategic combination of variable muzzle velocity and continuous fire mode ought to be employed. This method should be sustained until the target is either neutralized or forced to vacate the engagement area;

(3) In scenarios where the ammunition reserves aboard one's vessel are constrained, it

is imperative to concentrate engagement efforts at close range, extending up to the minimum firing range.

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