Teaching Research on Eigenvalues and Eigenvectors Based on the Fibonacci Number Sequence

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Abstract: Eigenvalues and eigenvectors are important concepts in linear algebra and have significant applications in artificial intelligence, big data analysis, and image processing. Most current textbooks introduce eigenvalues and eigenvectors directly from the perspective of mathematical definitions, which makes them difficult for beginners to understand. The difficulty in teaching eigenvalues and eigenvectors lies in making highly abstract mathematical concepts concrete and helping students understand their essence. To achieve the above objectives, this paper attempts to use the Fibonacci sequence, which is familiar to students, as an introductory example. It constructs a system of linear equations using the Fibonacci recurrence formula and applies linear algebra to study the Fibonacci sequence. With the goal of calculating the general term of the Fibonacci sequence, we naturally introduce the concepts of eigenvalues and eigenvectors, as well as the calculation methods for eigenvalues and eigenvectors. While solving problems, students acquire new knowledge. They understand the Fibonacci sequence from the perspective of linear algebra and deepen their comprehension of the concepts of eigenvalues and eigenvectors, as well as their geometric meanings.

Keywords: Eigenvalues; Eigenvectors; Geometric Meanings; Fibonacci Sequence.

1. Introduction

Eigenvalues and eigenvectors are important concepts in linear algebra. They apply previously learned content, such as determinants, systems of linear equations, and the structure of solutions to linear systems, and they serve as crucial tools for solving quadratic forms. Furthermore, the study of the eigenvalues and eigenvectors of matrices has significant applications in artificial intelligence, data analysis, and image processing [1-2].

Harold Hotelling proposed the principal component analysis (PCA) method based on eigenvalues and eigenvectors [3]. Since variables may have different dimensions, it is necessary to standardize each variable's data before using PCA; the covariance matrix is then used for analysis. The larger the eigenvalue of the covariance matrix, the more information the corresponding eigenvector contains. Conversely, the smaller the eigenvalue, the less information the corresponding eigenvector contains. PCA is commonly used for feature extraction, dimensionality reduction, face recognition, and image compression.

Chang Jingya and Wang Yijie introduced the application of eigenvalues and eigenvectors in magnetic resonance imaging (MRI) [4]. The principle of MRI is to infer microscopic tissue structures in the human body by measuring the diffusion motion of water molecules in confined spaces. The model used in the medical field to obtain images of internal tissues is called diffusion tensor imaging (DTI). This model uses a positive definite matrix known as the diffusion tensor at each voxel. The diffusion tensor characterizes the motion of water molecules in the human body. The eigenvector corresponding to the largest eigenvalue of the diffusion tensor indicates the primary direction of nerve fiber bundles.

Current textbooks often present the definitions of eigenvalues and eigenvectors directly and from a purely mathematical perspective. This makes it challenging for students who are newly exposed to this knowledge to grasp the concepts quickly. Teachers often focus only on explaining concepts and calculation methods in the teaching process. This results in students acquiring mathematical knowledge without the ability to apply it because they neglect the understanding of essential ideas such as eigenvalues and eigenvectors [5-7]. To help students understand the essence of eigenvalues and eigenvectors, as well as learn to apply mathematics to solve practical problems, Ma Lina and Liu Shuo explain these concepts using a combination of numerical and graphical methods [7]. This approach deepens students' understanding of the concepts. The authors also demonstrate the applications of eigenvalues and eigenvectors in real-life scenarios using two-dimensional principal component analysis (PCA) as an example. To help students intuitively understand the concepts of eigenvalues and eigenvectors, Guo Jinhai et al. presented animations illustrating the geometric meanings of these concepts [8]. They provided examples in two-dimensional and three-dimensional spaces respectively to observe the relationship between the original vectors and the transformed vectors, thereby deepening students' understanding of the concepts of eigenvalues and eigenvectors. Meanwhile, the corresponding properties of eigenvalues and eigenvectors were also introduced through the animations, enabling students to build new knowledge upon their existing understanding. To make the concepts of eigenvalues and eigenvectors more accessible to students of the humanities, economics, and management, Wei Xuan et al. introduced unit vectors from the perspective of land and labor as variables [9]. Other factors, such as science and technology and innovation capacity, can be represented by land and labor. Based on this, they introduced eigenvalues and eigenvectors. This teaching approach helps economics and management majors reflect on related professional content.

Although introducing or applying eigenvalues and eigenvectors from the perspective of principal component analysis (PCA) is practical, it requires students to learn a lot of background information, which may not pique their interest. This paper introduces the concepts of eigenvalues and eigenvectors through the Fibonacci sequence. This approach reduces the dryness of a direct introduction while using the concepts as tools to study the sequence, thereby deepening understanding.

2. Fibonacci Sequence

Leonardo Fibonacci introduced the Fibonacci sequence in the 13th century in his Liber Abaci (Book of Calculation), using the reproduction of rabbits in an ideal state as an example [10].

Rabbit Reproduction Problem:

(1) Initially, there is one pair of baby rabbits. Starting from the third month, they begin to reproduce one pair of baby rabbits each month.

(2) The newborn rabbits mature into adults and begin reproducing in their third month.

(3) This process continues indefinitely, assuming all rabbits survive and none die.

The sequence formed by the number of rabbit pairs each month is known as the Fibonacci sequence. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... We typically denote the general term of the Fibonacci sequence as $\{F_n\}$.

The Fibonacci sequence has many applications in daily life and aligns with numerous natural phenomena, such as the spiral patterns of seashells and the arrangement of sunflower seeds. A streaming media service system is a technical architecture that transmits multimedia content, such as audio and video, over networks. This allows users to play content in real time without fully downloading files. A common cache strategy used in these systems is static data scheduling technology. The Skyscraper Algorithm is a classic algorithm for solving such problems and is considered nearly perfect. The ordinary Fibonacci algorithm adopts a transmission channel division that does not account for increased server bandwidth to emphasize theoretical analysis. Building on this classic algorithm, Gong Zhuorong et al. proposed an improved scheme based on the Fibonacci sequence [11]. This scheme replaces the Fibonacci piecewise proportional sequence with the network model to improve delay control, reduce disk storage requirements, and achieve better network transmission.

Low-density parity-check (LDPC) codes are highly efficient, linear, block error-correcting codes that are widely used in communication and storage systems due to their excellent properties. However, storing the parity-check matrices of LDPC codes with random structures requires substantial storage space when the code length is large. Tanner and Fossorier built on LDPC codes to propose QC-LDPC (quasi-cyclic LDPC) codes. QC-LDPC codes can achieve linear-complexity encoding using simple shift registers; however, they are inflexible in terms of code parameter selection and thus fail to meet the needs of practical applications. Zhan Wei et al. proposed a method for constructing QC-LDPC codes based on the Fibonacci sequence [12]. This method's encoding algorithm exhibits linear complexity with respect to the code length, making it easy to implement.

The significance of Fibonacci sequence is so profound that there is an international journal, The Fibonacci Quarterly, dedicated solely to publishing research on its theoretical developments and practical applications. For the sake of convenience, this paper lists some of the most commonly used properties of the Fibonacci sequence.

Property 1: Recursive Relationship of the Fibonacci Sequence:

$$F_{n+2} = F_{n+1} + F_n, \quad (n \geq 0) \tag{1}$$

where $F_0 = 0$, $F_1 = 1$

Property 2: General Term of the Fibonacci Sequence:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right], n = 0,1,2,\cdots, \tag{2}$$

Property 3: The limit of the ratio of the general terms in the Fibonacci Sequence:

$$\lim_{n\to\infty}\frac{F_n}{F_{n+1}} = \frac{\sqrt{5}-1}{2}, \tag{3}$$

where $\frac{\sqrt{5}-1}{2}$ is the golden ratio.

3. Eigenvalues, Eigenvectors, and Their Geometric Significance

In this section, we will present the definitions, calculation methods, and physical significance of eigenvalues and eigenvectors. Most textbooks define eigenvalues and eigenvectors as follows.

Definition [13]: Let A be an *n*-order matrix. If there exists a scalar λ and a non-zero n-dimensional vector *x* such that

$$Ax = \lambda x. \tag{4}$$

Then, the scalar λ is called an eigenvalue of the matrix A, and the non-zero vector x is called an eigenvector of the matrix *A* corresponding to the eigenvalue *λ*.

In accordance with the definitions of eigenvalues and eigenvectors, the calculation methods for eigenvalues and eigenvectors are as follows:

Step 1: Calculate the characteristic roots $\lambda_i, i = 1,2,\cdots,k$ of the characteristic equation $|\lambda I - A| = 0$.

Step 2: Substitute each characteristic root λ_i into equation (4), and by combining terms, derive the homogeneous linear system of equations $(\lambda_i I - A)x = 0$.

Step 3: The solutions to the homogeneous linear system of equations $(\lambda_i I - A)x = 0$ are the eigenvectors corresponding to the characteristic value.

These methods can be used to find the eigenvalues and eigenvectors of low-order matrices, but they are not suitable for higher-order matrices. In Reference [4], Chang Jingya et al. introduced numerical methods for finding eigenvalues and eigenvectors, including the power method, inverse power method, and QR method. These methods can be used to find the eigenvalues and eigenvectors of large-scale matrices. Programming these methods to solve for the eigenvalues and eigenvectors of large-scale matrices can help students enhance their ability to apply mathematics in practice. However, due to the limited class hours generally allotted for linear algebra, this content can be supplemented with extracurricular materials.

Most textbooks rarely mention the geometric interpretation of matrices. Yong Longquan conducted a systematic study [5]. Using matrix invertibility and symmetry as classification criteria, he presented the geometric significance of eigenvalues and eigenvectors from the perspective of linear transformations, taking 2×2 matrices as an example. These classifications include invertible and non-invertible symmetric and asymmetric matrices. Due to the limited class hours for linear algebra and students' ability to comprehend the material, this content can be provided as supplementary material after class. Next, we present the geometric interpretations of eigenvalues and eigenvectors.

Geometric Meaning [5]: Let *A* be a matrix and $x \in R^n$. From the perspective of linear transformations, Ax represents performing a linear transformation on the vector *x*. An eigenvector is a vector that does not change direction or magnitude during a linear transformation. The transformation is equivalent to stretching or shortening the vector, and the corresponding scaling ratio, λ, is the eigenvalue.

4. Introducing Eigenvalues and Eigenvectors from the Perspective of the Fibonacci Sequence

The following section introduces eigenvalues and eigenvectors from the perspective of calculating the general term of the Fibonacci sequence. The property 3 of the Fibonacci sequence, i.e. $\lim_{n\to\infty}\frac{F_{n+1}}{F_n} = \frac{\sqrt{5}-1}{2}$, will be understood in depth in terms of the geometric meaning of eigenvalues and eigenvectors.

4.1. Calculating the General Term of the Fibonacci Sequence

There are many methods of calculating the Fibonacci sequence. One method is to construct a geometric sequence using the method of undetermined coefficients, the characteristic equation method, or the generating function method. The sequence can also be calculated from the perspective of eigenvalues and eigenvectors.

As Property 1 shows, the recurrence relation of the Fibonacci sequence is $F_{n+2} = F_{n+1} + F_n$. To facilitate application of matrices, we add an identity equation to rewrite it as:

$$\begin{cases} F_{n+2} = F_{n+1} + F_n \\ F_{n+1} = F_{n+1} \end{cases} \tag{5}$$

Eq. (5) can be written in the form of a matrix product:

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix}, \tag{6}$$

So,

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = \cdots = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n+1} \begin{pmatrix} F_1 \\ F_0 \end{pmatrix} \tag{7}$$

By the property of equality of matrices, to obtain F_n , it is only necessary to find $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{n+1}$, and then derive the result using matrix multiplication.

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, If there exists an invertible matrix P , such that $P^{-1}AP = \Lambda$, where $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$, then

$$A = P\Lambda P^{-1} \tag{8}$$

$$A^{n+1} = P\Lambda^{n+1}P^{-1} = P\begin{pmatrix} \lambda_1^{n+1} & 0 \\ 0 & \lambda_2^{n+1} \end{pmatrix}P^{-1} \tag{9}$$

This greatly simplifies the calculation of A^n .

The process of finding a matrix P such that $P^{-1}AP = \Lambda$ is called diagonalization of a matrix. Let $P = (P_1, P_2)$, where P_1, P_2 are the column vectors of matrix P, then $P^{-1}AP = \Lambda$ can be transformed into $(AP_1, AP_2) = (\lambda_1 P_1, \lambda_2 P_2)$. If the matrix A is diagonalizable if and only if there exist non-zero column vectors P_1, P_2 and scalars λ_1, λ_2 such that

$$AP_i = \lambda_i P_i, i = 1, 2 \tag{10}$$

Based on this, eigenvalues and eigenvectors are introduced. To solve for eigenvalues and eigenvectors, continuing to simplify equation (10) yields:

$$(\lambda I - A)P = 0 \tag{11}$$

Since equation (11) has non-trivial solutions, we set the determinant of its coefficients to zero: $|\lambda I - A| = 0$, Substituting A into the equation yields the eigenvalues $\lambda_1 = \frac{1+\sqrt{5}}{2}, \lambda_2 = \frac{1-\sqrt{5}}{2}$. Then we substitute λ_1, λ_2 into equation (11) to solve the homogeneous linear system, yielding the corresponding eigenvector $P_1 = \begin{pmatrix} \frac{\sqrt{5}+1}{2} \\ 1 \end{pmatrix}$, $P_2 = \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$. So,

$$A^{n+1} = P\Lambda^{n+1}P^{-1} = \begin{pmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{pmatrix}^{n+1} \begin{pmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{pmatrix}^{-1} \tag{12}$$

Where $P = \begin{pmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{pmatrix}$ and $\Lambda = \begin{pmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{pmatrix}$.

Simplifying equation (12) yields that

$$A^{n+1} = \frac{1}{\sqrt{5}} \begin{pmatrix} \left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2} & \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1} \\ \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1} & \left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n} \end{pmatrix} \tag{13}$$

Substituting Equation (13) into Equation (7) yields that

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} \left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2} & \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1} \\ \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1} & \left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} \left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2} \\ \left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1} \end{pmatrix} \tag{15}$$

So,

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n \right], n = 0, 1, 2, \cdots, \tag{16}$$

Using the method of eigenvalues and eigenvectors, we have not only derived the general term of the Fibonacci sequence but can also generalize this approach to solve the general term of $F_{n+2} = aF_{n+1} + bF_n$.

4.2. Understanding The Fibonacci Series in Terms of the Geometric Significance of Eigenvalues and Eigenvectors

Regarding two adjacent terms of the Fibonacci sequence as a two-dimensional array, that is, $(0,1), (1,1), (1,2), (2,3), \cdots, (F_n, F_{n+1}), \cdots$, and mapping them to points in the plane rectangular coordinate system, as shown in Figure 1.

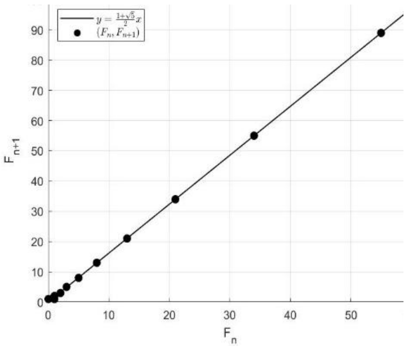

Figure 1. Scatter Plot of a Two-Dimensional Array (F_n, F_{n+1})

Through observation, we can see that these points are essentially on the same straight line. As n increases, the points get closer to the line. It is reasonable to assume that this line is $y = kx$.

When a linear transformation with $A=\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$ is performed on all integer points $(x, y)^{\mathrm{T}}$ in a rectangular coordinate system, it is observed that almost all of the points are rotated by the transformation. However, points on the line $y = kx$ are stretched but not rotated. From the geometric meaning of eigenvalues and eigenvectors, we know that the points on the straight line are eigenvectors of the matrix A, and that k is the corresponding eigenvalue.

The equation of the line is known from the eigenvalues: $y=\frac{1+\sqrt{5}}{2}x$. The points on a straight line are eigenvectors corresponding to eigenvalue $\lambda_1=\frac{1+\sqrt{5}}{2}$.

When $n\to\infty$, (F_n, F_{n+1}) will be on a straight line because

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\lim_{n\to\infty}\frac{\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]}{\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n}-\left(\frac{1-\sqrt{5}}{2}\right)^{n}\right]}=\frac{1+\sqrt{5}}{2} \quad (17)$$

In other words, (F_n, F_{n+1}) are eigenvectors corresponding to eigenvalue $\lambda_1=\frac{1+\sqrt{5}}{2}$. Since the eigenvalues represent the scaling ratios, we have $\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\frac{1+\sqrt{5}}{2}$, which leads to $\lim_{n\to\infty}\frac{F_n}{F_{n+1}}=\frac{\sqrt{5}-1}{2}$, thus verifying Property 3.

If we change the initial terms of the Fibonacci sequence to $F_0=2, F_1=1$ (without altering the recurrence relation $F_{n+2}=F_{n+1}+F_n$) and redraw the points (F_n, F_{n+1}), the result is shown in Figure 2.

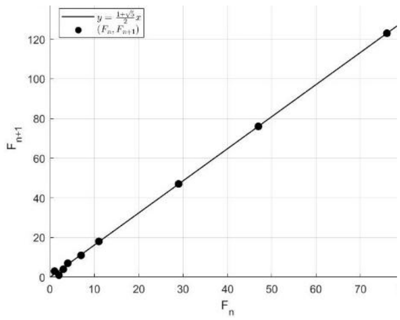

Figure 2. Scatter Plot of a Two-Dimensional Array (F_n, F_{n+1}) With Modified Initial Terms

As can be seen in Figure 2, the initial points are far from the straight line $y=\frac{1+\sqrt{5}}{2}x$. However, as n increases, points (F_n, F_{n+1}) gradually approach the line $y=\frac{1+\sqrt{5}}{2}x$. This is because although we changed the initial entries of the Fibonacci series, we did not change the transformation matrix $A=\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$, nor the corresponding eigenvalues and eigenvectors. Thus, as n increases, Figure 2 becomes similar to Figure 1.

5. Conclusion

Nature harbors countless magical phenomena, and mathematics is the key to understanding these mysteries. However, pure mathematical theories are often obscure and difficult to understand due to their abstract nature. When teaching the important concepts of eigenvalues and eigenvectors, teachers may wish to use the Fibonacci series, which students are familiar with. They can then naturally introduce the definitions and computation methods of these concepts through a step-by-step guide. This teaching design not only enables students to use the eigenvalue theory to solve the Fibonacci series, but also deepens their understanding of the nature of the series through the geometric intuition of eigenvectors. Through this process, students will gradually develop a unified mathematical mindset that integrates algebra and geometry. They will also cultivate dialectical thinking skills, enabling them to discern the essence beyond appearances.

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