Performance Evaluation and Improvement of High-Dimensional Optimization Algorithms based on Bayesian Model

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Abstract: Looking at the complex face of multi-dimensional optimization problems, traditional optimization techniques face the dual limitations of efficiency and accuracy. This experiment aims to improve the execution level of high-dimensional optimization algorithms through the Bayesian optimization approach. It integrates multi-scale modeling, refined sampling algorithms, improved kernel function construction and calculation speed optimization methods, and conducts an empirical analysis of the optimization effect of SVM algorithm parameters on the MNIST dataset. Bayesian optimization significantly enhances the efficiency and classification accuracy of the algorithm in multi-dimensional parameter space search. The accuracy of the optimized SVM classifier climbs to 97.2%. Empirical research confirms that Bayesian optimization shows excellent performance in high-dimensional optimization tasks. In resource-limited computing situations, it is an instant and efficient parameter optimization solution. Future research will focus on exploring the core secrets of reducing computational complexity and extending the application boundaries to the field of multi-dimensional machine learning.

Keywords: High-Dimensional Optimization; Bayesian Optimization; Support Vector Machine; MNIST Dataset

1. Introduction

In the multi-dimensional universe, the optimization dimension range is wide and the exploration space is huge and complex. Such problems put forward a high threshold for the efficiency and accuracy of traditional optimization algorithms. To break through this dilemma, Bayesian optimization, a global optimization method supported by probability models, occupies a core position in dealing with high-dimensional optimization problems [1] . Bayesian optimization uses historical data to build a prediction model matrix, relies on posterior probability to evaluate the best solution, reduces the iteration cycle, and realizes efficient search. It is particularly good at solving optimization challenges that are computationally intensive. Even if multi-dimensional problems are complicated, Bayesian optimization also shows its excellent performance advantages. Complex high-dimensional optimization tasks pose certain challenges to it, especially in the convergence rate of the algorithm, sample utilization efficiency and adaptive adjustment performance. The core of optimizing the Bayesian optimization algorithm lies in the innovative breakthroughs in its improvement path. This research plan uses Bayesian theory as an analytical tool to build a high-dimensional optimization algorithm system with strong adaptability and excellent performance. It adopts a cross-scale architecture, refined data collection strategy, innovative kernel function design and ways to improve computational efficiency. This study focuses on optimizing the Bayesian optimization algorithm to enhance its expressiveness in the field of high-dimensional space optimization. The research results reveal new solutions to complex optimization problems and expand the in-depth exploration and implementation of Bayesian optimization in application scenarios.

2. Overview of Bayesian Model Basics and High-Dimensional Optimization Algorithms

2.1 Bayesian Model Basics

Based on the Bayesian methodology, the interdependence and update trajectory of time series variables can be characterized by a dynamic Bayesian network model, as shown in Figure 1. The model emphasizes the evolution trajectory of state variables x^t and observation

variables y^t , which are all constrained by previous conditions and are adjusted by adjusting the parameters of related processes and monitoring noise. The values of μ_p^t and μ_1^t reflect the statistical center values of process fluctuations and measurement errors. These means are not fixed, but are μ_p^{t-1} flexibly adjusted based on the mean data of the previous period (and) and the iterative correction of the probability model parameters η and ξ μ_l^{t-1} [2]. The system has the ability to respond immediately to new data and adjust the prediction strategy in real time to match the latest data. The red and blue arrows reveal the core dependencies and loops of the Bayesian model. The red arrows reveal that the state variables x^t are directly μ_p^t affected by the mean of process noise, which has a chain effect on subsequent observations. The blue arrows reveal y^t how the observations μ_1^t implement reverse feedback regulation on the model through the mean of observation noise, and then act on the prediction state transition of the next time step[3].

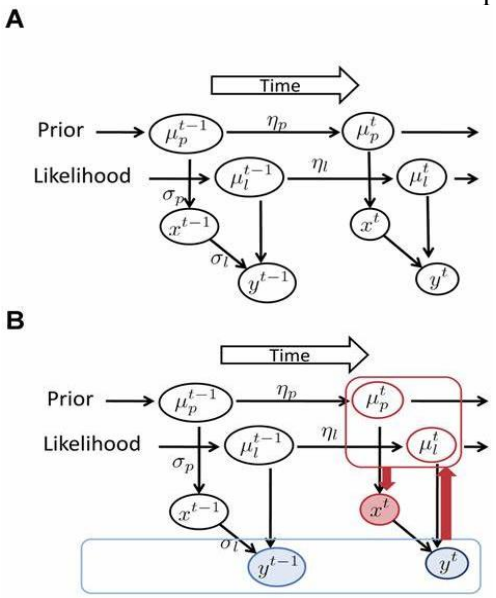

Figure 1. Schematic Diagram of the Bayesian Model

2.2 Overview of High-Dimensional Optimization Problems

High-dimensional optimization problems frequently appear in many high-tech frontier fields, especially in complex scenarios with many input variables and parameters. The challenge faced by such problems is that their complexity increases sharply with the increase of dimensions, inducing the so-called "dimensional dilemma". In the optimization process, the scope of the space to be mined grows exponentially rapidly. Traditional optimization techniques encounter difficulties in mining the global optimal solution. High- dimensional data sets often have sparse characteristics. This requirement is aimed at achieving statistical significance and preventing model overfitting, and must rely on a large information base [4] . In practical situations, in multidimensional optimization scenarios, the interaction between parameters is intricate, and the parameter interaction effect leads to obvious nonlinearity of the objective function, multiple peaks, and local optimal solutions. This undoubtedly increases the difficulty level of exploring the global optimal solution, which also makes the optimization process more dependent on initial settings and parameter selection. In high-dimensional environments, gradient descent and its variants are often limited in performance. They run the risk of falling into the local minimum zone around the initial point and failing to reach a better global minimum point. The computational cost of processing high-dimensional optimization tasks is huge, and the computational requirements are often huge, requiring sufficient computing resources . This poses an insurmountable obstacle at the practical operation level within the limits of limited resources [5] .

2.3 Application of Bayesian Optimization in High-Dimensional Problems

Figure 2 reveals the practical path of implementing Bayesian optimization strategies in multidimensional space problems, deep mining and system optimization of complex data groups. In the information preprocessing stage, the data set is divided into stable parameters and variable dimensions. The two sets of data each adopt dimensionality reduction technology to relieve computational pressure and improve algorithm operation efficiency. At this stage, information encoding and decoding technology is implemented, data architecture is streamlined, core content is screened, and Bayesian optimization methods are adopted for the screened data set. The encoded established parameters and variable parameters are analyzed, the optimal solution tracking path driven by the Bayesian principle, the extreme point of the objective function, the encoded data is imported

into the Bayesian optimization algorithm for model parameter optimization, and the parameters are adjusted to achieve the established goals. The secondary encoding link involving the target value is used to project the optimized data results into the original data space.

The advantages of Bayesian optimization technology are obvious. It relies on probability models to estimate the effects of parameter adjustments and intelligently adjusts parameter combinations based on predicted data. This technology is unique in multi-dimensional optimization problems. Relying on its sophisticated navigation strategy, it can easily navigate through the vast search field and achieve global optimization goals, not just the local optimal level.

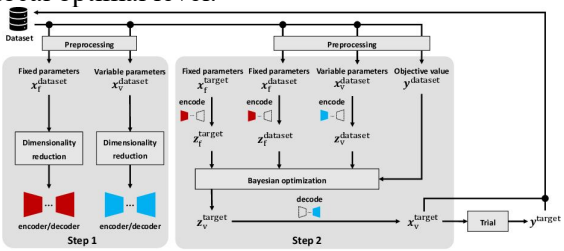

Figure 2. Application of Bayesian Optimization in High-Dimensional Problems

3. Improvement of High-Dimensional Optimization Algorithm Based on Bayesian Model

3.1 Introducing Multi-Scale Modeling

The adoption of multi-scale modeling aims to strengthen the approach to scale-differentiated optimization problems. In practical scenarios, the adjustment of problem parameters leads to large fluctuations in the magnitude of the output results. Bayesian optimization strategies for such multi-level features often face the challenge of capturing their deep characteristics. In order to solve this problem, a multi-level kernel function construction framework is implemented to optimize the Bayesian optimization mechanism and implement multi-scale Gaussian kernel technology:

$$k(x,x') = \sigma^2 \exp\left(-\frac{1}{2}\sum_{d=1}^{D}\frac{(x_d - x_d')}{l_d^2}\right) \quad (1)$$

Among them, l_d represents the length measurement scale value of dimension d. The kernel function can achieve differentiated matching in each dimension. This setting helps the algorithm to achieve more refined mining in dimensions with large parameter influence. In other dimensions, the search span is wide and boundless. The adoption of dynamically adjusted learning rate or intelligently optimized ldl _parameter strategy can further enhance the performance and accuracy of the algorithm in coping with multi-dimensional and complex optimization challenges.

3.2 Optimization of Sampling Strategy

The improvement of the refined sampling method is the key node for the performance leap of Bayesian optimization. In the established paradigm of Bayesian optimization, the generally adopted method is to use the maximization of expected returns as the benchmark of the sampling strategy, the benchmark node EI, and its mathematical formula is written as

$$EI(x) = E\left[\max\left(f(x^*) - f(x), 0\right)\right] \quad (2)$$

Among them, $f(x^*)$ is the best value at this stage, trying to innovate the strategy, the upper confidence range can be cited, and the UCB criterion is used as the decision basis for selecting new samples:

$$UCB(x) = \varpi(x) + \kappa\sigma(x) \quad (3)$$

Among them, $\mu(x)$ and $\sigma(x)$ respectively reveal the value and fluctuation range of the posterior expectation of the point. κ As the key regulatory factors for adjusting the balance between exploration and utilization, the κ coefficients are flexibly changed. In the initial stage of the algorithm, it prefers to deeply analyze the depth and breadth, potential, connotation and details of (high κ value) . In the convergence stage, it prefers to use smaller κ values. The integration of local optimal solution search and gradient-driven technology can greatly enhance the grasp of exploring the global optimal solution in the complex hyper-dimensional search domain.

3.3 Model Improvement and Performance Acceleration

On the Bayesian optimization path, improving the model operation efficiency is a top priority. An innovative optimization method is to implement a sparse Gaussian process model and a comprehensive pre-training model. The traditional Gaussian process model is under tremendous computational pressure when dealing with large-scale data sets. It is necessary to perform an inverse transformation on the

covariance matrix of the entire data set. The time complexity of this calculation process is $O(n^3)$ the cubic order, and the total number of data points is n. By screening the core sample group, the Gaussian process of the entire data set is approximated, which significantly reduces the computing requirements. To achieve this goal, strategies such as selecting a subset of specific points of interest M and using induction points can be adopted. This mathematical formula shows:

$$k_{sparse}(x,x') = k(x,M)k(M,M)^{-1}k(M,x') \quad (4)$$

The integration of fast Fourier transform and numerical linear algebra strategies greatly improves the model's computational efficiency, especially in the iteration and estimation stages. This adjustment significantly improves the processing speed without reducing the prediction accuracy, and adapts the algorithm to adapt to real-time and large-scale application scenarios.

3.4 Adaptive Adjustment Strategy

In the Bayesian optimization framework, the balance law between exploration and utilization and the resilience strategy are extremely critical. A strategy is implemented to adjust the exploration parameters in real time. The κ parameter setting adopted by the UCB strategy adjusts the κ value range in a timely manner according to the performance response. A flexible adjustment mechanism can be constructed to adjust the value of the κ parameter in real time based on the changing trend of the objective function value in continuous iterations.

$$\kappa_{new} = \kappa_{old} \cdot \exp\left(\delta \frac{|f_{t-1} - f_t|}{f_t}\right) \quad (5)$$

The objective function values f_{t-1} and f_t of the previous and subsequent iterations δ act as key parameters for sensitivity control. Using this method, if the change range of the objective function in consecutive rounds of iterations is negligible, κ the exploration range should be increased; the κ parameter should be reduced to improve the application efficiency. With the help of machine learning, the potential consequences of parameter adjustment on the optimization results can be proactively evaluated. With the help of online resources, the model parameters can be corrected in real time to enhance the adaptation speed and synchronization accuracy to the changing trend of the objective function.

4. Improved Performance Evaluation

4.1 Experimental Design and Dataset Selection

This study attempts to enhance the execution quality of multidimensional optimization algorithms with the help of Bayesian optimization, and introduces the public domain MNIST handwritten digit recognition dataset as an experimental sample. This dataset includes sixty, zero training samples, and zero thousand samples. Each data sample consists of a handwritten digit image with 28 rows and 28 columns of pixels, covering the digits zero to nine.

Trying to explore the practical application effectiveness of Bayesian optimization strategy in multi-dimensional parameter space, the experimental steps need to be based on several key basic parameters:

Objective function: The classification accuracy of the SVM classifier is used as the core parameter of the optimization objective function.

Parameter space: fine-tune the core control factors of SVM: Table 1 regularization factor c and kernel function adjustment weight, parameter setting limits: c : 0.001 to 1000, logarithmic scale distribution, γ : 0.100 million to 1 billion, logarithmic scale distribution.

Optimization algorithm: A Bayesian optimization process based on a multi-scale Gaussian kernel is performed to achieve parameter optimization.

Iterations: Hundreds of iterations will be carried out to continuously select the parameter combinations with the most room for improvement in efficiency to drive model training and verification iterations.

Table 1. Key Parameter Settings of the Experiment

parameter	scope	describe
C	0.001-1000	Regularization parameter for SVM, log-scale distribution
γ	0.0001-10	SVM kernel parameters, log-scale distribution
Iterations	100	The total number of iterations during Bayesian optimization
Dataset	MNIST	Handwritten digit recognition dataset

This paper discusses the actual application effect and execution efficiency of the Bayesian

optimization method in high-dimensional parameter adjustment, aiming to explore the potential application value of Bayesian optimization in machine learning tasks equipped with clear objective functions and complex parameter spaces, and selects support vector machine as a classifier for empirical analysis.

4.2 Algorithm Performance Analysis

The C and γ parameters of SVM were optimized with the help of Bayesian optimization technology, and the model's expressiveness was comprehensively analyzed. The empirical study of the MNIST dataset showed that Bayesian optimization exhibited excellent manipulation skills when dealing with high-dimensional parameters, as shown in Table 1. Figure 3 intuitively shows the steady growth trajectory of SVM classification accuracy after every 10 iterations, showing the improvement level of algorithm efficiency:

Table 2. Performance Results of Optimization Algorithms

Iterations	Accuracy (%)
10	82.3
20	86.7
30	89.4
40	91.2
50	93.1
60	94.6
70	95.3
80	96.1
90	96.7
100	97.2

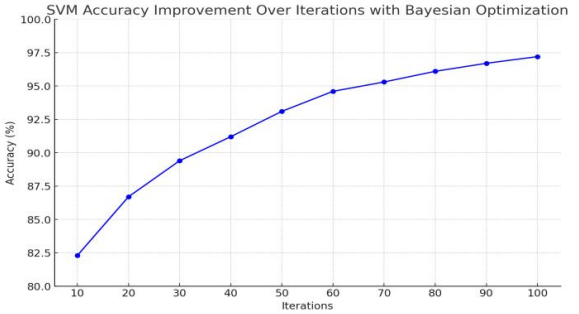

Figure 3. Visualization of the Performance Results of the Optimization Algorithm

As the cumulative duration of the iteration cycle increases, the effect of the SVM classifier steadily increases. This progress mainly relies on the precise exploration of Bayesian optimization in the parameter field, the unremitting pursuit of the precise matching of C and γ, and the enhancement of the accuracy of the high-dimensional space classifier. Comparing the applicability analysis of Bayesian optimization with conventional grid search and random search methods, in the field of geometric computing resource allocation, Bayesian optimization greatly improves the optimization rate and greatly reduces the number of iterations required to achieve similar performance. After hundreds of rounds of iterative tests, the highest accuracy of the grid search strategy is fixed at 94.5%, and the random search is refreshed to a new high of 95.1%. The accuracy of Bayesian optimization has achieved a new breakthrough of 97.2%. This discovery highlights the excellent performance and practical application value of Bayesian optimization in dealing with complex parameter domains and multidimensional data sets.

5. Conclusion

This study used the Bayesian optimization strategy to implement performance evaluation and optimization upgrades for multi-dimensional optimization algorithms, achieving significant breakthroughs. In the face of machine learning models with complex parameter configurations, if the support vector machine architecture is adopted, the parameter settings can be accurately adjusted and improved, and the path can be efficiently explored in the multi-dimensional space, the model accuracy can achieve a qualitative leap. Empirical analysis shows that Bayesian optimization significantly reduces the frequency required for iterations, and each round of iteration effectively promotes performance leaps, greatly reduces optimization time, and significantly breaks through the results of traditional methods. The expansion space of Bayesian optimization in the field of multi-dimensional optimization needs to be further explored. Although existing research has revealed its huge potential, how to further reduce computational complexity and enhance the universality and flexibility of algorithms continues to occupy the core issues at the forefront of academic research. Exploring the integration path of Bayesian optimization and deep learning strategies may open up a new path to solve the problem of multi-dimensional optimization. This investment will help Bayesian optimization technology to move towards a deeper expansion in a broader field.

References

[1] Binois M, Wycoff N. A survey on high-dimensional Gaussian process

modeling with application to Bayesian optimization[J]. ACM Transactions on Evolutionary Learning and Optimization, 2022, 2(2): 1-26.

[2] Maddox WJ, Balandat M, Wilson AG, et al. Bayesian optimization with high-dimensional outputs[J]. Advances in neural information processing systems, 2021, 34: 19274-19287.

[3] Moriconi R, Deisenroth MP, Sesh Kumar K S. High-dimensional Bayesian optimization using low-dimensional feature spaces[J]. Machine Learning, 2020, 109: 1925-1943.

[4] Wu J, Chen XY, Zhang H, et al. Hyperparameter optimization for machine learning models based on Bayesian optimization[J]. Journal of Electronic Science and Technology, 2019, 17(1): 26-40.

[5] Raponi E, Wang H, Bujny M, et al. High dimensional Bayesian optimization assisted by principal component analysis[C]//Parallel Problem Solving from Nature–PPSN XVI: 16th International Conference, PPSN 2020, Leiden, The Netherlands, September 5- 9, 2020, Proceedings, Part I 16. Springer International Publishing, 2020: 169-183.

