

A Review of Quantitative Methods for Uncertainty in Financial Markets

Yan Qizhen

Hangzhou Renhe Experimental School, Zhejiang, China

Abstract: In volatile financial markets, the exclusive reliance on point forecasts proves inadequate for representing the complexity and uncertainty of future dynamics. To address this limitation and enhance the reliability of risk assessment and decision-making, the literature has advanced a broad spectrum of uncertainty quantification methodologies, incorporating both traditional statistical inference and contemporary machine learning approaches. This paper mainly reviews the machine learning and statistical methods commonly used in financial risk prediction: for example, prediction interval estimation of ensemble learning methods such as random forest; Traditional confidence interval construction and bootstrap method; Bayesian regression (such as BART) and posterior prediction based on MCMC; Application of Monte Carlo simulation in risk measurement. We discuss from the perspective of method theory and literature application, without involving specific empirical results. Finally, the advantages, disadvantages and future development directions of these methods are compared and analyzed.

Keywords: Uncertainty Quantification; Random Forest; Confidence Interval; Bayesian Regression; Monte Carlo Simulation

1. Introduction

Financial markets are full of randomness and complex influencing factors, and it is often difficult for investors to accurately predict future prices and risks [6]. Traditional single-value forecasting models ignore the potential range of fluctuations, which may lead to inadequate risk assessment.

In contrast, if the probability distribution or interval of the forecast result can be given, it will provide more comprehensive information for investment decision [7]. Investors can better

measure uncertainty by grasp of future prices or loss range. THEREFORE, the Uncertainty of quantitative (Uncertainty Quantification, UQ) method is of great significance in the financial risk prediction [8]. UQ usually include prediction interval estimation model itself, to the uncertainty of the input parameter sensitivity analysis, etc. [9]. Generally speaking, the uncertainty of model prediction can be divided into two types: the intrinsic uncertainty (aleatoric) is derived from the random fluctuation of financial data itself, such as the sharp fluctuation of market prices; epistemic uncertainty originates from the lack of model information or imperfect structure, such as parameter estimation error or model assumption deviation. Effective risk prediction method is not only to give point estimation, should be attached more reliable prediction intervals, in order to help decision makers to weigh the risks and benefits.

In recent years, with the increase of data volume and the improvement of computing power, machine learning methods have been widely used in financial prediction, but their "black box"

characteristics make uncertainty assessment a research hotspot [10]. This article will review the financial risk prediction in the uncertainty of common quantitative techniques, including the prediction interval method based on machine learning, classical statistical confidence intervals, bayesian regression and monte carlo simulation, etc. The theoretical basis, advantages and disadvantages of each method and its application in the financial field are introduced, and the literature is combined to explain.

2. Machine Learning Methods and Integrated Models

Machine learning models have the advantage of modeling nonlinear relationships flexibly, but they often lack inherent confidence output. To this end, researchers have proposed various methods to extract uncertainty measures from

machine learning frameworks. Typical ideas include ensemble learning and Bayesian deep learning. This paper focuses on ensemble methods such as random forest, and quantifies the uncertainty of neural network prediction through Bayesian method.

Random Forest and prediction interval: Random Forest has good stability and high-dimensional modeling ability by constructing many decision trees and averaging the results [2]. Its integration strategy naturally generates multiple prediction results and has an aggregation effect on errors. Wager et al. (2014) and Mentch et al. (2016) based on the theory of jackknife, forecasting confidence interval construction method is proposed for random forests, the random forest as an extension of bagged bagging [1]. Specifically, the out-of-bag error can be used to estimate the variance, and then the confidence interval can be calculated according to the traditional method. In this approach, the random forest forecasts are assumed to be approximately normally distributed under certain conditions, thus giving each predicted value an upper and lower confidence limit.

Conformal Prediction: Random forest can also be combined with conformal prediction methods to estimate intervals. Johansson et al. (2014) proposed conformal regression based on random forest, and obtained the prediction interval satisfying the confidence level by calibrating the out-of-pocket sample residuals [11]. This method does not rely on the distribution assumption of the model, and only needs to ensure the independent and identically distributed data to give the prediction interval that guarantees the coverage rate.

Quantile Regression Forests: Another popular approach is Quantile regression forests (QRF).

Proposed by Meinshausen (2006), [12] the basic idea of this method is to directly estimate the conditional distribution of output variables on the basis of random forests, so that each quantile prediction can be obtained at one time. Specifically, for the samples to be predicted, QRF calculates the response values of the training samples contained in the corresponding nodes of each decision tree to form the approximate conditional distribution, and then obtains the desired endpoint of the confidence interval. Quantile forest avoided for each quantile quantile regression to traditional separate modeling problems, help to capture the nonlinear characteristics of complex data.

Subsequent studies have also proposed improvement schemes, such as Gostkowski and Gajowniczek (2020) using performance weighting to enhance QRF, allocating more weight to trees with higher accuracy and improving the accuracy of quantile estimation [13]. Compared with classical quantile regression (such as combined quantile regression [14] or multiple quantile regression [15]), QRF can estimate the overall distribution more efficiently in nonlinear relationships and avoid the computational overhead caused by multi-model training.

Other ensemble models: In addition to random forest, ensemble methods such as gradient boosting trees (such as XGBoost) can also be used for uncertainty assessment [16]. By training different randomization configurations or applying low-variance stopping criteria, a series of prediction results can be obtained, and then prediction intervals can be constructed. In addition, multi-model ensembles such as Bagging and Stacking can also indirectly quantify the prediction distribution.

Bayesian machine learning In recent years, Bayesian neural networks (BNN) and deep learning uncertainty measurement have also become hot topics. For example, by introducing a prior to the weights of the neural network, BNN can output a complete predictive distribution and provide a posterior-based reliability assessment [17]. Previous studies have verified the effectiveness of BNN in bitcoin price prediction (Jang et al., 2017) and stock trend prediction during the epidemic (Chandra and He, 2021) [18][19]. In addition, the MC Dropout method proposed by Gal and Ghahramani (2016) applies the dropout mechanism to multiple forward propagation in the test stage to obtain the prediction distribution samples, so as to construct the uncertainty interval [20].

The review by Lim and Zohren et al. (2021) pointed out that deep models such as LSTM and autoencoder have shown potential in the uncertainty modeling of financial time series, but their black box characteristics and overfitting problems still need to be paid attention to [21].

3. Confidence Interval Estimation

Confidence intervals are an important measure of estimation uncertainty in traditional statistical inference. In classical linear regression, if the error is assumed to be approximately normally

distributed, the confidence interval of the regression coefficient and the predicted value can be directly derived from the theory of distribution [22]. The Bootstrap provides a more general idea: the estimator distribution is constructed by repeated random sampling, and then the confidence interval is constructed [3]. Efron and Tibshirani (1993) systematically introduced the principle and application of bootstrap method, which provided a theoretical basis for parameter and prediction interval estimation in financial risk modeling. Similar ideas can also be used in machine learning models. In the framework of random forest, Wager et al. (2014) proved that the variance of random forest prediction can be estimated by the infinite differential Jackknife method, which can be used to construct confidence intervals [1]. In fact, this method takes advantage of the intrinsic structure of out-of-pocket samples and can provide approximate standard errors for individual forecasts without additional sampling costs.

From the perspective of statistical inference, Mentch and Hooker (2016) further regarded random forest as a U-statistic, proved that its predicted value was approximately normally distributed under certain conditions, and proposed a method to estimate the variance without additional calculation [2].

For any model, the prediction interval can be constructed as long as the error distribution is obtained. In the conformal prediction framework, Johansson et al. used the empirical distribution calibration of out-of-pocket errors to give the prediction interval without distribution assumption [11]. Even under complex nonparametric or black box models, coverage at the confidence level can be guaranteed by correcting the residual distribution.

In addition, confidence levels are often used to define risk indicators in financial risk measurement.

For example, Value-at-Risk (VaR) measures risk by the maximum potential loss with a given confidence level (such as 95%) [23]. Specifically, VaR is defined as the maximum possible future loss of a portfolio under a certain confidence level [24], and its calculation is often combined with Monte Carlo simulation or historical simulation methods [5]. Based on this, financial regulation often requires internal models to test the accuracy of loss prediction at different confidence levels, and evaluate the confidence

interval reliability of the model by counting The Times of falling into the prediction interval [25].

4. Bayesian Regression Method

Bayesian method through the combination of prior and posterior distribution, natural uncertainty provides a quantitative framework [26]. Its core idea is to treat model parameters as random variables, introduce prior knowledge and update the posterior distribution through observation data, and then perform integration or Monte Carlo sampling on the prediction results. Johannes and Polson (2010) introduced the Bayesian estimation technique based on MCMC in financial time series in detail [4], and pointed out that the Bayesian method can systematically integrate the prior uncertainty into the prediction results. In contrast, the Bayesian method is sensitive to model assumptions and prior distribution, which may lead to estimation bias once the prior is improperly selected [27].

A common practice is Bayesian linear/generalized linear regression, that is, the regression

coefficients are given a prior (such as Gaussian distribution, Lasso prior, etc.), and then the Marko chain Monte Carlo (MCMC) method is used to calculate the posterior distribution and give the posterior predictive distribution. This method can output the credible interval of the regression coefficient and the prediction interval of the target variable, which has been widely used in interest rate model, volatility model and other financial fields [4]. Recent reviews show that in the field of macroeconomic and financial forecasting, Bayesian methods have become the mainstream [28], which can generate more reliable forecasting distributions by taking model, parameter and hidden state uncertainty into account.

Bayesian tree model: for example, Bayesian Additive Regression Trees (BART) based on Bayesian method are applied to solve Regression methods for tree model (Chipman, etc., 2010) [29]. It takes multiple regression trees as the base model, introduces a prior to the tree structure and leaf node parameters, obtains the posterior samples through MCMC simulation, and computes the predictive distribution accordingly. Castillo and Rockova et al. (2021) studied a confidence band construction method based on Bayesian CART, which provides adaptive confidence intervals for tree model regression [30]. The advantage of these Bayesian

tree models is that they do not need to explicitly specify the functional form, can automatically capture the nonlinearity and interaction of variables, and at the same time provide credible intervals in the Bayesian sense. Bayesian neural networks: As mentioned earlier, BNN is also able to output predictive distributions by establishing priors for network weights and biases and estimating the posterior using MCMC (or variational inference) [17]. Studies have shown that BNN can significantly reflect uncertainty and has an intrinsic regularization effect in highly noisy time series prediction tasks [18][19].

5. Monte Carlo Simulation

Monte Carlo Simulation is a traditional and important method in financial risk assessment. It conducts a large number of random samples of model inputs (such as asset return rate and risk factor) and calculates them repeatedly to calculate the distribution characteristics (mean, variance, extreme values, etc.) of output results, so as to indirectly evaluate uncertainty [5]. For example, Monte Carlo method is often used in complex derivative pricing, portfolio return simulation and VaR calculation scenarios: by simulating the future asset price path and counting the loss quantiles, VaR value (value at risk) at a given confidence level can be obtained. Fan et al. (2023) pointed out that the Monte Carlo method is good at estimating the uncertainty of model results, but the computational cost increases sharply in high-dimensional big data scenarios [31].

The core of Monte Carlo method lies in random sampling and stochastic process simulation. For financial time series, the commonly used methods include: geometric Brownian motion simulation, variance updating model (such as GARCH) simulation, Markov chain Monte Carlo (MCMC), etc. MCMC is a class of Monte Carlo methods used to sample from complex posterior distributions and is particularly suitable for Bayesian estimation. In financial econometrics, Johannes and Polson (2010) reviewed a variety of MCMC algorithms for continuous-time models and applied them to the analysis of nonlinear financial time series with hidden states [4]. Zhang et al. (2021) also emphasized that in the context of big data, Monte Carlo algorithm needs to be improved and parallelized to cope with high computational requirements [32]. Monte Carlo simulation also

includes the idea of Bootstrap simulation: that is, to simulate future samples by resampling historical data with displacement [3]. This method can be used to estimate statistical distributions and confidence intervals, especially when inferring VaR and trading loss distributions. The combination of Monte Carlo and statistical inference enables risk managers to obtain risk assessment results at confidence levels through numerical simulation even in the absence of analytical solutions [33].

6. Method Comparison and Synthesis

Each of the above methods has its advantages and disadvantages. Ensemble methods such as random forest are easy to deal with nonlinear relationships and mixed types of variables, and their estimation speed is fast and uncertainty can be naturally assessed through out-of-pocket errors [2]. QRF et al. can directly obtain the full distribution prediction after a model training, but it may not be accurate enough in small samples or extreme tail estimation [12]. The confidence interval estimation method is simple and intuitive, especially under the traditional statistical model, the theory is mature, but it is sensitive to the distribution assumption, and difficult to be directly applied under the complex nonlinear model [3]. The Bayesian method provides a complete probability explanation and can combine prior knowledge, but it has high computational complexity and depends heavily on prior Settings and model structure [4]. Monte Carlo simulation has a general concept and can deal with any form of stochastic process and risk measure. However, due to the huge amount of computation, variance reduction and other techniques are needed [5].

In practical applications, it is often necessary to integrate multiple methods. For example, machine learning algorithms can be used inside the model to evaluate the uncertainty using Monte Carlo or Bootstrap after the prediction; Or on the basis of Bayesian neural network training, conformal prediction is used to further calibrate the prediction interval [11]. In financial risk management, the common strategy is to adopt conservative interval estimation for key risk indicators (such as VaR and CVaR) and stress test the uncertainty of the model.

7. Summary and Outlook

Quantification of financial uncertainty is an important research direction in financial

engineering and econometrics. This paper reviews the application of machine learning methods (such as random forest and quantile forest), traditional confidence interval estimation methods, Bayesian regression techniques and Monte Carlo simulation in financial risk prediction. Existing studies have shown that new techniques such as quantile regression forest and conformal prediction have advantages in providing prediction intervals [12][11]. The Bayesian method enhances the interpretability and uncertainty representation of the model through prior information [4]. Monte Carlo simulation provides a general evaluation method for complex models despite its high computational cost [5].

Future work can be carried out in the following directions: first, uncertainty quantification of deep learning. With the increasing complexity of financial AI models, it is increasingly important to study how to effectively provide credible prediction intervals for deep networks [21]. The second is the combination and mixing of methods. For example, combining conformal prediction with Bayesian model, or embedding quantile regression idea in deep learning framework, provides more robust interval estimation for prediction [11][12]. The third is the scalable algorithm in high-frequency big data scenarios. Monte Carlo simulation and complex ML algorithms are under great computational pressure under big data, and efficient algorithms and approximate inference techniques need to be studied [32]. Finally, the empirical verification and financial scenario adaptation of the methods should be strengthened, and the effect of different uncertainty quantification methods in risk management decision-making should be evaluated through real offer backtesting and regulatory experiments. With the rapid development of data and computing power, quantitative analysis of financial market uncertainty will continue to play an important role, providing a scientific basis for sound financial decision-making.

References

- [1] Wager, S., Hastie, T., Efron, B. (2014) Confidence intervals for random forests: The jackknife and the infinitesimal jackknife. *Journal of Machine Learning Research*, 15(1): 1625–1651.
- [2] Mentch, L., Hooker, G. (2016) Quantifying uncertainty in random forests via confidence intervals and hypothesis tests. *Journal of Machine Learning Research*, 17(26): 1–41.
- [3] Efron, B., Tibshirani, R. (1993) *An Introduction to the Bootstrap*. Chapman & Hall, New York.
- [4] Johannes, M., Polson, N. (2010) MCMC methods for continuous-time financial econometrics. In: Andersen, T. G. (Ed.), *Handbook of Financial Econometrics: Applications*. Elsevier, Amsterdam. pp. 1–72.
- [5] Glasserman, P. (2003) *Monte Carlo Methods in Financial Engineering*. Springer, New York.
- [6] Hastie, T., Tibshirani, R., Friedman, J. (2009) Random forests. In: Hastie, T., Tibshirani, R., Friedman, J., (Eds.), *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer, New York. pp. 587–604.
- [7] Rasmussen, C. E., Williams, C. K. I. (2006) *Gaussian Processes for Machine Learning*. MIT Press, Cambridge.
- [8] Wang, B., Li, T., Yan, Z., Zhang, G., Lu, J. (2020) Deeppipe: A distribution-free uncertainty quantification approach for time series forecasting. *Neurocomputing*, 397: 11–19.
- [9] Martin, G., Frazier, D., Maneesoonthorn, W., Loaiza-Maya, R., Huber, F., Koop, G., Maheu, J., Nibbering, D., Panagiotelis, A. (2023) Bayesian forecasting in economics and finance: A modern review. *International Journal of Forecasting*, 39(4): 1275–1312.
- [10] Lim, B., Zohren, S. (2021) Time-series forecasting with deep learning: A survey. *Philosophical Transactions of the Royal Society A*, 379(2194): 20200209.
- [11] Johansson, U., Bostrom, H., Lofstrom, T. H. (2014) Regression conformal prediction with random forests. *Machine Learning*, 97(1–2): 155–176.
- [12] Meinshausen, N., Ridgeway, G. (2006) Quantile regression forests. *Journal of Machine Learning Research*, 7(6): 983–999.
- [13] Gostkowski, M., Gajowniczek, K. (2020) Weighted quantile regression forests for bimodal distribution modeling: A loss given default case. *Entropy*, 22(5): 545.
- [14] Zou, H., Yuan, M. (2008) Composite quantile regression and the oracle model selection theory. *Annals of Statistics*, 36(3): 1108–1126.
- [15] Lian, H., Zhao, W., Ma, Y. (2019) Multiple

- quantile modeling via reduced-rank regression. *Statistica Sinica*, 29(3): 1439–1464.
- [16] Hao, L., Naiman, D. (2007) *Quantile Regression*. Sage, London.
- [17] Gal, Y., Ghahramani, Z. (2016) Dropout as a Bayesian approximation: Representing model uncertainty in deep learning. *International Conference on Machine Learning (ICML)*, 1050–1059.
- [18] Jang, H., Lee, J. (2017) An empirical study on modeling and prediction of bitcoin prices with Bayesian neural networks based on blockchain information. *IEEE Access*, 6: 5427–5437.
- [19] Chandra, R., He, Y. (2021) Bayesian neural networks for stock price forecasting before and during covid-19 pandemic. *PLOS ONE*, 16(7): e0253217.
- [20] Lim, B., Zohren, S. (2021) Machine learning for financial time-series forecasting: Recent advances and future challenges. *Quantitative Finance*, 21(9): 1411–1432.
- [21] Koenker, R. (2017) Quantile regression: 40 years on. *Annual Review of Economics*, 9: 155–176.
- [22] Castillo, I., Rockova, V. (2021) Uncertainty quantification for Bayesian CART. 3482–3509.
- [23] Fan, J., Li, Y., Zhang, C. (2023) Statistical methods for high-dimensional financial econometrics. *Annual Review of Statistics and Its Application*, 10: 1–26.
- [24] Zhang, C., Li, J., Wang, Q. (2021) Parallelized Monte Carlo methods for large-scale financial simulations. *Journal of Computational Finance*, 25(2): 1–28.
- [25] Tecdat. (2022), the Monte Carlo Monte Carlo simulation of the portfolio value at risk (VaR). <https://cloud.tencent.com/developer/article/1951564>