Stability Analysis of Finite-time Trajectory Tracking for Quadrotor Unmanned Aerial Vehicles Based on Backstepping Method

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Abstract: Quadcopter drones have been widely applied in fields such as aerial photography, logistics, and environmental monitoring due to their compact structure and strong maneuverability. However, its nonlinear and strongly coupled dynamic characteristics pose challenges to trajectory tracking control. The backstep method is a nonlinear control approach based on the Lyapunov stability theory, which achieves system stability by recursively designing virtual control quantities. This paper focuses on the application of the backstep method in the finite-time trajectory tracking of quadrotor unmanned aerial vehicles (UAVs), and conducts an analysis from four dimensions: theoretical framework, stability conditions, robustness optimization, and future development directions, aiming to provide theoretical support for improving the trajectory tracking accuracy and stability of UAVs in complex environments.

Keywords: Quadcopter Unmanned Aerial Vehicle; Reverse Step Method; Finite-time Trajectory Tracking; Stability Analysis; Nonlinear Control

1. Introduction

1.1 Research Background and Significance

Quadcopter unmanned aerial vehicles achieve vertical takeoff and landing as well as three-dimensional maneuverability through the speed difference of the four rotors, but their dynamic models have underactuation, strong coupling and nonlinear characteristics [1]. For instance, the attitude changes of unmanned aerial vehicles (UAVs) can affect position control through the rotation matrix, and position errors need to be corrected through attitude adjustment, thus forming a complex nonlinear coupling relationship. In logistics and distribution, drones need to land precisely on dynamic platforms within a limited time. In disaster rescue operations, it is necessary to complete trajectory tracking by traversing complex air flow environments. Due to the simplified model and fixed parameters, traditional PID control is difficult to cope with such scenarios [2]. The backstep method can effectively handle nonlinear coupling problems by decomposing the system into position and attitude subsystems and designing virtual control quantities step by step, providing a theoretical tool for finite-time trajectory tracking [3].

1.2 Current Research Status at Home and Abroad

The current trajectory tracking control methods for quadrotor unmanned aerial vehicles mainly include PID control, sliding mode control, model predictive control (MPC), and backstepping method. PID control is widely used in industrial scenarios due to its simple structure, but it is prone to overshoot and oscillation under wind disturbance or model uncertainty [4]. Sliding mode control enhances robustness by designing sliding mode surfaces, but high-frequency switching is prone to cause chattering [5]. MPC enhances accuracy through rolling optimization, but it has a high computational complexity and is difficult to meet real-time requirements [6]. Since the backstep method was proposed in the 1990s, it has gradually been applied in the field of unmanned aerial vehicle (UAV) control. For instance, Yuhong Hou et al. [7] proposed a backstep control based on the dynamic in-plane model method. By introducing a dynamic surface, it avoids the expansion of virtual control quantity calculation and achieves asymptotic tracking of position trajectories. However, most existing studies focus on asymptotic stability, with insufficient analysis of finite-time convergence, and have not fully explored the impact of model uncertainty on stability [8].

2. Backstep Theoretical Framework and Quadcopter Unmanned Aerial Vehicle Modeling

2.1 Basic Principles of the Reverse Step Method

The core idea of the backstep method is to decompose a complex nonlinear system into multiple subsystems and achieve overall stability through recursive design of virtual control quantities. Take a second-order nonlinear system as an example. Suppose the system state is composed of two variables, and the control objective is to make one of the state variables track the preset reference signal. The design process first defines the error variable as the difference between the actual state variable and the reference signal, and ensures the stability of the first subsystem by constructing virtual control quantities (such as speed control quantities). Subsequently, a new error variable is defined as the difference between the second state variable and the virtual control quantity, and the actual control input (such as the acceleration control quantity) is designed. Finally, the asymptotic stability of the entire system in the global range is proved through the Lyapunov stability theory. This method transforms the stability problem of high-order systems into a series of stability problems of low-order subsystems by eliminating error terms step by step, and is particularly suitable for systems like quadrotor unmanned aerial vehicles that have strong coupling and nonlinear characteristics.

2.2 Construction of Dynamic Model for Quadcopter Unmanned Aerial Vehicle

The dynamic model of a quadcopter unmanned aerial vehicle consists of two core parts: position dynamics and attitude dynamics. Position dynamics describes the translational motion of unmanned aerial vehicles (UAVs) in an inertial coordinate system. Based on Newton's second law, it can be expressed as: the rate of change of the position vector is jointly affected by the rotation matrix (converting the airframe coordinate system to an inertial coordinate system), the total thrust generated by the four rotors, the acceleration due to gravity, and external disturbances (such as wind force). Attitude dynamics describes the rotational motion of unmanned aerial vehicles (UAVs) around the three axes of the airframe coordinate system (roll, pitch, and yaw). According to the Euler equation, the rate of change of the angular velocity vector is closely related to the airframe inertia matrix, control torque (generated by the difference in rotor speeds), and skew-symmetry matrix (reflecting the coupling effect of angular velocity and inertia product). In the model, attitude angles (roll angle, pitch angle, yaw angle) directly affect the generation of position control instructions through the rotation matrix, forming a typical nonlinear coupling relationship. For instance, when a drone needs to move from its current position to the target point, it must first change its flight direction by adjusting the pitch angle and roll angle, and then achieve the position change by regulating the total thrust. This process demonstrates the strong coupling characteristics of position and attitude.

2.3 Applicability of Backstepping Method in the Control of Quadcopter Unmanned Aerial Vehicles

The backstep method can effectively handle the nonlinear coupling problem of the quadrotor unmanned aerial vehicle system by decomposing it into the position subsystem and the attitude subsystem. In the position control layer, a virtual speed control quantity is designed to converge the position error to zero. In the attitude control layer, the virtual velocity control quantity is converted into the expected attitude Angle (such as pitch Angle, roll Angle), and then the attitude control law is designed to make the actual attitude track the expected value. This method eliminates errors step by step, avoiding the complexity of directly dealing with high-order nonlinear systems while ensuring the global stability of the system. For instance, in circular trajectory tracking tasks, the backstep method can decompose the trajectory into radial position control and tangential velocity control: radial position control maintains the distance between the unmanned aerial vehicle and the center of the circle by adjusting the pitch angle and roll angle; tangential velocity control achieves uniform circular motion by adjusting the total thrust; and ultimately, smooth trajectory tracking is achieved through attitude adjustment. In addition, the backstepping method can enhance the system's robustness against parameter uncertainties and external disturbances by introducing integral terms or adaptive laws, further improving the control performance of quadcopters in complex environments.

3. Analysis of Stability Conditions for Finite Time Trajectory Tracking

3.1 Definition and Criteria for Finite-time Stability

Finite-time stability emphasizes that the system state converges to the equilibrium point within the preset finite time and remains stable during subsequent operation. This contrasts with the traditional asymptotic stability (where the state approaches the equilibrium point infinitely but does not necessarily reach it within a finite time). For nonlinear systems, the determination depends on constructing a continuously differentiable Lyapunov function: this function takes a value of zero at the equilibrium point and is positive at other state points; Meanwhile, its time derivative must meet specific conditions, that is, as the state deviates from the equilibrium point, the derivative shows a negative value and the decay rate is fast enough. In the trajectory tracking of quadrotor unmanned aerial vehicles (UAVs), it is necessary to design control laws so that both the position error (the deviation between the actual position of the UAV and the expected trajectory) and the attitude error (the deviation between the actual attitude angle and the expected attitude angle) meet the above conditions. For instance, the position control law can be constructed by introducing a nonlinear feedback term (such as the fractional power of the error) to form a Lyapunov function, and by adjusting the control parameters, the position error can be reset to zero within a finite time, thereby ensuring that the unmanned aerial vehicle can quickly track the target trajectory.

3.2 Proof of Stability Under Backstep Control

Take the position subsystem as an example. First, define a Lyapunov function based on position error. Its derivative can be derived through the position dynamics equation (describing the law of position change of the unmanned aerial vehicle) and the virtual velocity control law (intermediate control quantity, used to connect position and attitude control). By designing the actual control torque through the attitude control layer, the actual speed of the unmanned aerial vehicle can track the virtual speed, ensuring that the derivative of the Lyapunov function meets the stability condition (i.e., the derivative is negative and the attenuation speed meets the requirements). By further introducing finite-time control terms (such as nonlinear feedback or dynamic compensation), the derivative of the Lyapunov function can be modified to directly satisfy the finite-time stability condition (i.e., the error returns to zero within a finite time). Similarly, for the attitude subsystem, a Lyapunov function based on attitude error is defined. By deriving its derivative and introducing a finite-time control strategy, the finite-time convergence of attitude error can be proved. Ultimately, by combining the stability analysis results of the position and attitude subsystems, the overall finite-time stability of the trajectory tracking of the quadrotor unmanned aerial vehicle can be ensured.

3.3 The Influence of Parameter Selection on Stability

The selection of control gain and finite-time parameters directly affects the convergence speed and robustness of the system. Increasing the control gain can accelerate error convergence, but an overly large control gain can lead to saturation of the control input and trigger oscillation. Another control gain is used to suppress overshoot, and its value needs to match the former. The finite-time parameter determines the nonlinearity of the convergence speed. The smaller the parameter, the faster the convergence, but the more sensitive it is to the uncertainty of the model. For instance, in a wind-disturbed environment, if the parameters are selected too small, minor model errors may lead to system instability. Therefore, parameters need to be adjusted through simulation and experiments to strike a balance between convergence speed and robustness.

4. Robustness Optimization Strategy Controlled by Backstepping Method

4.1 Analysis of External Disturbance Suppression Capacity

Quadcopter drones are vulnerable to external disturbances such as wind and air currents during flight. For instance, crosswind can cause a drone to shift its position. If the control law does not take disturbances into account, errors will continue to accumulate. The backstep method can enhance robustness by introducing an interference observer. A disturbance observer is designed to estimate the disturbance and compensate for it in the control law. It is proved through the Lyapunov function that if the observation error is bounded, the system can still achieve finite-time stability. For instance, under crosswind conditions, the backstep control method using an interference observer can cause the position error to converge to a smaller range within a short period of time, while the uncompensated control error continues to increase.

4.2 Research on Model Uncertainty Adaptability

The parameters of the quadcopter unmanned aerial vehicle model may be uncertain due to load variations or manufacturing errors. The sensitivity of the backstep method to model parameters can be reduced through adaptive control. An adaptive law is designed to update the parameter estimates. It is proved through the Lyapunov function that if the parameter estimation error is bounded, the system can still remain stable. For instance, when the mass of a drone increases due to carrying additional loads, adaptive backstepping control can restore the trajectory tracking error to a small range within a short period of time, while the error of non-adaptive control continues to increase.

4.3 Composite Control Strategy under Multi-source Interference

In actual flight, unmanned aerial vehicles (UAVs) may simultaneously encounter multiple sources of interference such as wind disturbance, model uncertainty, and sensor noise. The compound control strategy enhances robustness by integrating the backstepping method, sliding mode control and adaptive control. For instance, by designing a reverse-sliding mode hybrid control law and adjusting the parameters, the requirement for model accuracy can be reduced while ensuring the stability within a limited time. Simulation shows that when wind disturbance and mass uncertainty coexist, the compound control strategy can make the trajectory tracking error converge to a small range in a short time, which is significantly better than the single backstep method control.

5. Future Development Directions and Challenges

5.1 Integration of Intelligent Control Methods

With the development of artificial intelligence technology, the integration of deep learning, reinforcement learning and backstepping methods has become a trend. For instance, deep neural networks can be utilized to estimate model parameters or disturbances online, replacing traditional interference observers. Optimize control gain through reinforcement learning to achieve adaptive parameter adjustment. Experiments show that the auxiliary backstep control method can expand the uncertainty adaptation range of the model, while the optimized control gain can shorten the convergence time.

5.2 Multi-Machine Collaborative Trajectory Tracking Control

Quadcopter unmanned aerial vehicles need to achieve synchronous trajectory tracking of multiple aircraft in tasks such as formation flight and collaborative search. The backstep method can achieve coordination by designing distributed control laws. For instance, each unmanned aerial vehicle (UAV) designs a local control law based on the backstepping method, and at the same time shares neighbor state information through a consensus protocol to adjust the control input to maintain formation. Simulation shows that in the formation flight of multiple unmanned aerial vehicles (UAVs), the distributed backstepping method control can make the formation error converge to a small range in a short time and is robust to communication delay.

5.3 Enhanced Adaptability in Complex Environments

In complex environments such as indoor spaces and urban canyons, there are problems like magnetic interference and GPS signal occlusion, which lead to a decline in the accuracy of trajectory tracking based on traditional inertial navigation. The backstepping method can be combined with technologies such as visual navigation and LiDAR SLAM to enhance adaptability. For instance, the visual odometer is used to estimate the pose of the unmanned aerial vehicle (UAV), and the visual error is introduced into the backstepping control law to achieve trajectory tracking in an environment without GPS. Experiments show that in an indoor environment, the vision-assisted backstep control method can keep the trajectory tracking error within a small range and within a small value.

6. Conclusion

The backstep method provides an effective theoretical tool for finite-time trajectory tracking of quadrotor unmanned aerial vehicles by recursively designing virtual control quantities. This paper conducts an analysis from four dimensions: theoretical framework, stability conditions, robustness optimization, and future development directions, revealing the advantages of the backstep method in handling nonlinear coupling, suppressing external disturbances, and adapting to model uncertainties. In the future, with the integration of intelligent control methods, the development of multi-aircraft collaborative control technology and the improvement of adaptability to complex environments, the reverse step method will play a greater role in the control field of quadcopter unmanned aerial vehicles (UAVs), promoting the wide application of UAVs in logistics, rescue, monitoring and other fields.

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