Single-Degree-of-Freedom Spring-Mass System: Linear Theoretical Framework, Nonlinear Extensions, and Paradigm Evolution

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Abstract: The single-degree-of-freedom (SDOF) spring-mass system is one of the most basic idealized models in structural dynamics. This paper shows how the SDOF theoretical framework has evolved from the classical analytical theory based on Newtonian mechanics and linear assumptions to the numerical-computation paradigm for capturing physical reality via nonlinear constitutive relations, and finally to an emerging integrated-systems framework that couples real-time sensing with active control theory. Using a decomposition framework organized around the system's fundamental physical attributes (damping, stiffness, and inertia), the paper systematically explains how nonlinear damping, dynamic-fracture mechanisms, and the introduction of the inerter fundamentally change dynamic behavior and lead to changes in modeling paradigms. The analysis indicates that the research paradigm has moved from seeking closed-form analytical solutions to finding a balance between computational tractability and physical fidelity, and is now moving toward the construction of intelligent systems marked by a modeldatacontrol closed loop. The paper ends by systematically identifying the core challenges and conceptual gaps left within the current theoretical landscape.

Keywords: Single-Degree-of-Freedom System; Linear Vibration; Nonlinear Dynamics; Damping Model; Inerter; Research Paradigm; Review

1. Introduction

The axiomatic basis of knowledge in vibration engineering and structural dynamics is the theoretical framework of the single-degree-of-freedom (SDOF) spring-mass system. The value of its theory does not only depend on its mathematical simplicity, but also it serves as a whole frame of reference that fully shows the basic principles of linear system dynamics and gives an essential theoretical standard to its own extensions into the nonlinear realm.

This classical linear theoretical system is based on two basic assumptions to make it predictive: first, the constitutive relation of the material follows Hooke Law, i.e. the restoring force is linearly related to displacement; second, the energy dissipation mechanism of the system can be well modeled by a viscous damping model that is linear in velocity. Both of these conditions ensure that the governing equation of the system is a linear ordinary differential equation and therefore the principle of superposition holds and consequently the system can be fully analyzed in the frequency domain as well as analytically solved [1, 2].

Nevertheless, in engineering practice, physical systems are often non-Hookean (e.g. material yielding, gaps) and non-viscous (e.g. dry friction, material hysteresis). The existence of these nonlinear mechanical behaviors causes systematic discrepancies between the predictions of linear theory and physical reality. Therefore, the main driving force behind research on SDOF systems is the constant improvement of their force-displacement and force-velocity relationships to build mathematical models that can better predict and describe real physical processes. This refinement process essentially reflects a transition from a closed, fully analytically solvable linear theoretical system to an open, generalized dynamics model system that depends on numerical computation and experimental validation [3, 4].

The paper seeks to answer the following questions in a systematic way: What theoretical benchmark and applicability boundaries does the linear theoretical framework of the SDOF system and its inherent assumptions establish? In order to overcome the limitations of this benchmark, what important theoretical extensions and model modifications have been triggered across the three fundamental physical dimensions that govern system dynamics-damping, stiffness, and inertia? What new computational and verification challenges do these extensions introduce as they improve model predictive capability? Finally, how do these different extensions collectively point towards a future development framework integrating modeling, system identification, and active control?

In order to answer these questions, the paper will be organized in the following way: First, it strictly derives the linear governing equation and specifies its conditions of validity and theoretical limitations. Second, it explains the inherent laws of linear systems by analyzing free and forced vibration. Then, using a decomposition framework based on physical attributes, it critically reviews forefront research in nonlinear damping, nonlinear stiffness, and inertial extensions. Finally, through comprehensive comparison and gap analysis, this paper argues that SDOF system research is undergoing a methodological revolution from system analysis to system design [4, 5]. Its core objective is changing from understanding and predicting the dynamic behavior of existing systems to actively endowing systems with new, intended dynamic characteristics. The future breakthrough is in resolving the systemic mismatch between the modeling, data, and control components [4].

This paper is a critical review and not an original experimental research. It seeks to synthesize, assess and contextualize current theories and models of SDOF systems thus explaining the evolutionary path of the system, identifying the current core issues, and future directions.

2. Physical Modeling and Linear Theoretical Framework: Benchmark and Boundary Conditions

Consider an idealized SDOF system, with its dynamics characterized by a mass block mm, a linear spring (stiffness k), a viscous damper (damping coefficient c), and possible external excitation F(t), as shown in Figure 1. Taking the static equilibrium position as the origin, the system displacement is represented by x(t). Based on Newton’s second law, the system dynamics are described by the following equation:

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=F(t) \quad (1)$$

This is the mathematical basis of classical linear vibration theory. The analytical solution is complete only if the principle of linear superposition is valid, which in turn is based on the fundamental assumption that the internal forces (spring restoring force and damping force) are linearly proportional to the system state variables (displacement and velocity).

The strength of this linear theoretical framework is that it provides a specific, fully solvable theoretical standard. Nevertheless, the creation of this standard also clearly defines its applicable theoretical limits. This limit is the collection of all dynamic problems meeting the linear assumptions.

In addition, classical linear theory has been rigorously established to show that static deformation does not affect the natural frequency of an SDOF system [1, 2]. Chen (2017) showed that when the spring axis at a state of equilibrium is tangent to the vibration path and the generalized mass does not vary with the coordinate, the natural frequency remains $\omega_n=\sqrt{\frac{k}{m}}$, irrespective of the amount of static displacement. This finding strengthens the theoretical limit of the linear model since it demonstrates that the modal parameters and dynamic response are independent of the pre-deformation in the linear assumptions [6].

Once the relationship between force-displacement or force-velocity of the system is no longer linear, such as when restoring force has nonlinear stiffness of the form, or damping force has non-smooth properties like Coulomb friction, then the governing equation becomes a nonlinear differential equation and the superposition principle fails, and hence the completeness of its analytical solution also fails. Therefore, the fundamental importance of developing the linear theoretical system is not only to solve problems in its boundary but also to provide a performance comparison and theoretical starting point for all nonlinear problems beyond this boundary [2].

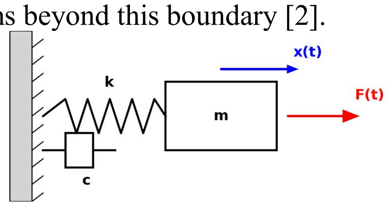

Figure 1. Schematic Diagram of the Single-degree-of-freedom System, should Include Clearly Labeled m, k, c, F(t), and Displacement x(t)

Figure 1 Caption: Idealized model of a single-degree-of-freedom (SDOF) spring-mass-damper system.

3. Free and Forced Vibration: Intrinsic Laws and Observable Phenomena in Linear Systems

The system will experience free vibration when the external excitation $F(t)=0$. Its transient response properties are defined by the eigenparameters of the system. When there is no damping ($c=0$), the system will be undergoing undamped simple harmonic motion at the natural frequency $\omega_n=\sqrt{\frac{k}{m}}$, which is a property of the system. Upon introducing viscous damping ($c>0$) the nature of the return to equilibrium of the system is determined by the dimensionless damping ratio $\zeta=\frac{c}{2\sqrt{mk}}$, and it can be underdamped ($\zeta<1$) decaying oscillation, critically damped ($\zeta=1$) fastest non-oscillatory or overdamped ($\zeta>1$) slow return [1, 2], as illustrated in Figure 2.

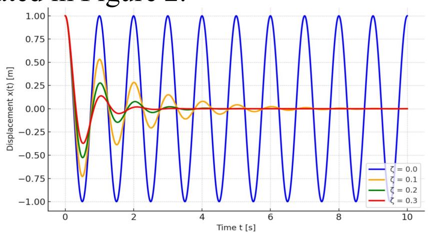

Figure 2. Free Vibration Response Curves of Various Damping Ratios

Figure 2 Caption: Time-varying free vibration response of an SDOF system at varying damping ratios ζ.

When the system is subjected to harmonic excitation $F(x)=F_0\cos\omega t$, its steady-state response is a harmonic function of the same frequency. The amplitude $X(\omega)$ and phase lag $\phi(\omega)$ determined by the system's frequency response function:

$$X(\omega)=\frac{\frac{F_0}{k}}{\sqrt{(1-(\frac{\omega}{\omega_n})^2)^2+(2\zeta\frac{\omega}{\omega_n})^2}},\phi(\omega)=\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-(\frac{\omega}{\omega_n})^2}\right) \quad (2)$$

The resonance is observed when the frequency of the excitation ω gets close to the natural frequency of the system ω_n and the amplitude of the response becomes the largest. The damping ratio ζ directly regulates the bandwidth of the system and sharpness of the resonance peak [1, 2], as shown in Figure 3.

$$\text{Normalized amplitude } \left(\frac{X}{\frac{F_0}{k}}\right) \quad (3)$$

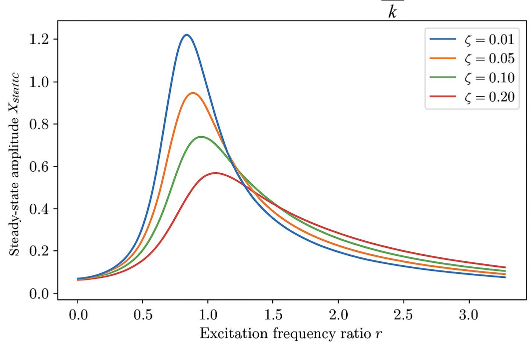

Figure 3. Amplitude-frequency Response Curves of Forced Vibration at Various Damping Ratio

Figure 3 Caption: Normalized steady-state amplitude of a forced SDOF system as a function of frequency ratio $r=\frac{\omega}{\omega_n}$ under different damping ratios ζ. The amplitude is normalized by the static displacement $\frac{F_0}{k}$.

It should be noted that the entire image of resonance presented above is valid only under the assumption of linear systems. In nonlinear systems, the frequency response characteristics are fundamentally different; amplitude-dependent natural frequency, response jumps, superharmonic and subharmonic resonances can occur [3]. In such cases, the frequency response curve given by linear theory is no longer a valid approximation of the system behavior and may give an entirely incorrect physical picture. Table 1 summarizes the specific effects of these parameters on the SDOF system behavior.

Table 1. Summary of Parameter Effects on SDOF System Behavior

Parameter change	Natural frequency ω_n	Damping ratio ζ	Free vibration decay	Resonance frequency	Resonance peak	System bandwidth
m ↑	↓	↓	Slower	↓	↑	Narrower
k ↑	↑	↓	Faster	↑	↓	Wider
c ↑	—	↑	Faster	Slight ↓	↓	Wider

4. Nonlinear Frontier Extensions: Theoretical Revisions Based on Physical Attribute Decomposition

In order to estimate physical reality, the force relations of the linear model should be re-examined. This part breaks down and discusses forefront extension work in terms of three basic physical properties that determine system dynamics: damping (dissipation), stiffness (restoration), and inertia (kinetic energy storage).

4.1 Damping Model Extensions: From Linear Approximation to Characterization of Non-

Smooth Dissipation Mechanisms

The linear viscous damping model is a convenient mathematical approximation of the more complex energy dissipation mechanisms, which is physically distorted. The difference between its experimental measurement and theoretical description presents one of the main challenges in using the SDOF model in precision machinery and contact dynamics. Marino and Cicirello (2020) [7] experimentally measured the steady-state response and stick-slip motion boundary under Coulomb friction with a precision-controlled SDOF system. This work has theoretical importance because it empirically proves that any linear model cannot predict dynamic behaviors caused by non-smooth nonlinear forces, such as distorted frequency-response curves and abrupt phase hysteresis.

But the modeling of damping mechanisms at the engineering frontier has gone beyond classical Coulomb-friction models. Fractional calculus models, by introducing a convolution relationship between stress and strain, provide the mathematical foundation for non-local, memory-dependent damping. Research shows that for complex media like viscoelastic materials, using fractional derivatives (intermediate between pure elasticity and pure viscosity) to represent their constitutive relations achieves much higher accuracy [5]. When dealing with non-smooth or memory-dependent dynamic behaviors, research methods have to move from analytical solutions to numerical simulation. As a result, numerical time integration and fractional differential equation solvers are essential tools.

Besides material-based and interface-based damping mechanisms, recent research has revealed that nonlinear damping can also arise from the geometric configuration itself. The study by Kuttan et al. (2024) [8] shows that adding a secondary spring to dynamically control the normal force on a frictional interface results in strongly nonlinear energy dissipation. Their model attained an almost 40% improvement in effective damping efficiency and got rid of the amplitude—jump phenomena usually related to geometric nonlinearity. A particularly significant finding is that when the second spring is at its vertical equilibrium position, the system displays its lowest natural frequency and strongest off-resonance attenuation, which suggests an optimal setup for energy dissipation. This work brings attention to an important but overlooked mechanism: the coupling among geometry, normal-force modulation, and nonlinear friction, expanding damping research beyond traditional frictional and viscoelastic models.

The contemporary theoretical problem is that there is no general damping scheme that would be able to integrate the various dissipation mechanisms, including rate-independent Coulomb friction and displacement-dependent hysteresis, memory-based fractional models, and geometry-based nonlinear friction. The current approximate analytical methods like equivalent linearization and harmonic balance method are trade-offs between computational efficiency and physical fidelity under certain operating conditions. As damping theory deepens—from linear viscosity to non-smooth friction and beyond to memory-based fractional and geometry-modulated models—it shows a basic limitation: making damping models more physically accurate makes them mathematically and computationally more complex. This universal rule gives a fundamental premise for model selection and validation in nonlinear dynamics [3].

4.2 Stiffness Model Extensions: From Continuum Mechanics to a Zero-Dimensional Mechanism-Isolation Framework

The use of the SDOF formulation in dynamic fracture problems shows that it has a unique epistemic value as a zero-dimensional mechanism-isolation model. By removing all spatial degrees of freedom from the system, it reveals the temporal dynamics that control failure processes without interference from geometric or field-gradient complexities. This is what differentiates the SDOF framework from classical vibration analysis. In classical analysis, model reduction mainly aims at computational convenience. In dynamic fracture, its role is fundamentally conceptual—to isolate inertia-driven temporal effects that are otherwise included in full continuum models.

The recent work by Kazarinov et al. (2024) [9] is a good illustration of the explanatory power of this very simplified modeling paradigm. Their research demonstrates that even a linear mass-spring oscillator supplemented with a rate-sensitive or softening-type failure criterion can reproduce two typical phenomena reported in high-rate fracture experiments:

Rate-dependent apparent strength increase.

Under rapidly rising loads, inertia gives rise to a temporal deformation lag. This means the oscillator can't reach its quasi-static displacement corresponding to the instantaneous load. The lag functions as an inertia barrier, allowing the system to survive loads beyond its static strength. The resulting "apparent strengthening" is not a property of the material but an emergent effect from the competition between loading rate and inertial timescale.

Fracture delay under pulse-type loading.

When the system is subjected to a short-duration pulse, fracture can happen after the external load has already reached its peak. This delay is usually explained in continuum mechanics through nonlocal temporal functionals (e.g., incubation-time criteria). It naturally comes from the SDOF oscillator as a result of inertial accumulation of elastic energy. The oscillator effectively "remembers" the previous stress history even though the load has decreased.

These findings prove that the SDOF oscillator is a temporal model of more sophisticated continuum fracture models [9]. The effects, which are usually ascribed to microstructural kinetics or complicated field evolution (dynamic increase in fracture toughness or crack initiation delay), can be qualitatively described by an inertial system. This strengthens the opinion that inertia does not just modify the stress distribution but is one of the main factors causing dynamic failure on short time scales.

Nevertheless, the basic drawback of this modeling method should be underlined. Due to the fact that the SDOF model is naturally zero-dimensional, it is not capable of depicting any spatially distributed phenomena: crack-tip stress intensity factors, energy-release rates, cohesive-zone evolution, crack-path instability, microbranching, stress-wave interactions, or localization patterns. As a result, the time-dependent measures of the oscillator (e.g., delay time, dynamic strength) are conceptually incommensurate with continuum fracture measures such as dynamic fracture toughness or crack-growth laws. They cannot be directly compared numerically because they are at different ontological levels of physical description.

The dimensional discontinuity between lumped-parameter and continuum models must be acknowledged explicitly, as well as mathematically scaled, to bridge these frameworks. Validation should be based on systematic cross-comparison with full-field numerical methods (e.g., XFEM, cohesive-zone FE, peridynamics) or high-speed experimental diagnostics (e.g., DIC, caustics), which can resolve crack-tip fields and propagation dynamics.

Overall, the SDOF model is a mechanism-isolation platform in dynamic fracture mechanics. Its worth is not to make quantitative predictions but to explain the dominant temporal effects-inertia-governed strengthening and delay—before spatially resolved theories are used. When interpreted correctly, these simplified models take an essential place in the multi-scale fracture analysis hierarchy, acting as a conceptual base for more advanced formulations to be systematically built upon.

4.3 Inertial Element Extensions: Introduction of the inerter and Reconstruction of system transfer functions

The invention and use of the inerter is a paradigm shift as it introduces a new two-terminal inertial element, instead of modifying existing parameters to actively reconfigure dynamic characteristics of the system [10].

Basili et al. (2019) [11] have shown that the attachment of an inerter to a spring-damper system in parallel can add new anti-resonance poles to the frequency response function of the SDOF system, and consequently provide effective vibration suppression at certain frequencies.

It is also interesting to note that the idea of SDOF model does not only involve new elements; as a simplified and abstract design philosophy, it has great potential in controlling vibrations of more complex structures. As an example, one method used in vibration suppression studies of rod-coupled systems is optimizing parameters to make the dominant modal dynamic properties of a complex continuous subsystem equal to those of a nonlinear SDOF system for control strategy design. This equivalent SDOF control philosophy demonstrates its conceptual simplicity and design effectiveness in handling multi-degree-of-freedom systems [12].

The physical nature of the inerter is that it gives the system an equivalent inertial path independent of the mass, thereby changing the modal energy distribution of the system. This represents a shift in SDOF system design from parameter optimization within a given structure (m, c, k) to actively shaping its transfer function by introducing new physical elements or equivalent control strategies.

The core scientific question in the future is how to develop a co-design theory for the inerter and semi-active/active control elements, such as magnetorheological dampers, and how to scale the SDOF control philosophy more effectively to large, complex structures. This requires developing new control laws that consider displacement and velocity feedback, along with the dynamic states introduced by the inerter or equivalent SDOF state in the feedback loop. The aim is to achieve truly adaptive vibration control across broad frequency bands [4, 11].

5. Comprehensive Discussion: Paradigm Evolution and Theoretical Frontiers

A systematic literature review on SDOF system research shows that there has been a paradigm shift in the field. This change is not an incremental accumulation of research but a paradigm shift in research philosophy and methodology, which focuses on redefining the relationship between model accuracy, physical reality, and engineering applicability.

5.1 Structural Transformation of Research Paradigms

The intellectual development of SDOF system research is represented as a series of different methodological paradigms, each of which redefines the relationship between model fidelity, analytical tractability and engineering utility. The first Linear Analytical Paradigm created the basis of the field, its predictive power being based on Newtonian mechanics and a linear mathematical framework. This paradigms main contribution was to provide closed-form solutions that gave full spectral and temporal characterization of system response, albeit under the strict and often physically limiting assumptions of Hookean elasticity and viscous damping [1, 2].

The ubiquitous experience of nonlinearities material yielding, friction, geometric nonlinearities led to a paradigm shift in the direction of Numerical Computation. This was a radical re-alignment between mathematical convenience and physical authenticity. The central task became numerical integration of governing equations and use of specialized analyses, such as experimental investigations of nonlinear oscillators with dry friction [7], and numerical algorithms for fractional derivative models [3, 5]. The recognition of numerical solutions as the main results of research indicated a new balance that gave priority to computational efficiency and phenomenological correctness over the completeness of analytical solutions.

The rise of this numerical paradigm is not only demonstrated but also critically complexified by seminal contributions made by the domestic scholarship, which explain its inherent methodological tensions in an outstanding manner. In the field of nonlinear damping optimization, the research conducted by Tian (2025) [13] offers a stark contrast to the global use of iterative numerical schemes. When global studies have mainly used equivalent linearization [7] or genetic algorithms—approximate methods—to deal with the indeterminate forms that arise in DVA optimization, Tian uses L'Hôpital's rule to obtain a closed-form analytical solution. This method does not simply estimate but strictly gets the best damping condition, setting up a theoretical standard. Its deep limitation, though, is that it depends axiomatically on system differentiability, a requirement that isn't met in regimes where impact or dry friction exists, thus showing a fundamental limit of the analytical approach in a numerical age.

Meanwhile, in the field of intelligent control, the study by Zhong et al. (2022) [14] touches a critical epistemological debate in control theory: the difference between model-based and model-free paradigms. In contrast to popular international approaches such as Adaptive Sliding Mode Control or Active Disturbance Rejection Control (ADRC), which attempt to improve performance by improving physical modelling or explicit disturbance estimation, Zhongs use of Full-Form Dynamic Linearization Model-Free Adaptive Control (FFDL-MFAC) is a more radical step. It eliminates the model-identification step by surgery. It shows that precise stabilization of a magnetically levitated system (as shown by a 0.005 decrease in overshoot and a 0.2607 decrease in RMS displacement error versus PID) can be achieved solely from input-output data streams. The resulting epistemological cost is a black-box controller. Its operational success is not linked to mechanistic understanding, raising reasonable concerns about its extrapolative reliability in untested operational areas.

In addition to these applied developments, the study by Li, Li and Zhang (2025) [15] re-establishes the role of SDOF system as a laboratory of mechanistic discovery. Their numerical analysis of a bilaterally restricted impact oscillator, revealing such basic sequences as grazing bifurcations and period-doubling paths to chaos is a direct offspring of the global tradition of research into piecewise-smooth dynamical systems. This book does not merely apply; it gives the basic vocabulary of nonlinear phenomena, the bifurcations and attractors that form the basis of the behaviour of real-world discontinuous systems, thus providing the conceptual ground on which applied control and optimization techniques are eventually constructed.

Together, these domestic researches solidify the central antinomy of the modern numerical paradigm: a strong and frequently essential improvement in application-specific predictive capacity is acquired at the cost of advanced numerical processing and a common sacrifice of generalizable physical understanding. They show that the move to numerical computation is not only a tool change, but an overall transformation of the nature of dynamical inquiry. A detailed comparison of these international and domestic research directions is provided in Table 2.

Table 2. Comparison of Research Directions (International vs Domestic)

Research Direction	Representative Work (International)	Core Contribution/ Mechanism	Representative Work (China)	Domestic Focus
Nonlinear damping	Marino & Cicirello (2020)	Experimental Quantification of Coulomb friction, stick-slip motion boundaries	Tian, Q. K. (2025)	Contrasts with Contrasts with international numerical methods (e.g., equivalent linearization): Provides a unified analytical solution for optimal damping using L'Hôpital's rule, offering exact results versus common approximations.
Dynamic fracture	Kazarinov et al. (2024)	Demonstrates rate-dependent dynamic strength; fracture delay; mass-on-spring model as surrogate for dynamic fracture	Limited domestic studies on high-rate fracture	Conforms to the international basic research: Offers mechanistic study of bifurcations and chaos in collision systems, generalizing the work on non-smooth dynamics.
Vibration control	Basili et al. (2019)	Inerter introduces anti-resonance; enhances broadband vibration suppression	Zhong, Z., Cai, Z., & Qi, Y. (2022)	Contrasts with international model-based strategies (e.g., ADRC): It uses a data-driven, model-free adaptive control (MFAC) paradigm, which does not require explicit modeling of the system and identification of its parameters.

Now, the discipline is indicating tendencies of an emerging Intelligent Integration Paradigm. This new paradigm attempts to transcend traditional disciplinary boundaries and build a complete research structure that incorporates physical modeling, system identification, and active control. Its most fundamental aspect is the change in the nature of passively characterizing the behavior of systems into actively designing and controlling dynamic response. In this paradigm, even complex dynamic representations such as fractional models can be implemented in real-time controllers to obtain intelligent control through accurate model prediction [4, 11].

5.2 Key Challenges and Theoretical Frontiers

Against the backdrop of paradigm evolution, current research faces three interconnected core challenges:

First, at the theoretical level, there is the issue of fragmented constitutive models. The current nonlinear models are mainly empirical formulas for specific phenomena, and they do not have a unified theoretical framework based on fundamental mechanical principles. For example, although empirically successful, the physical mechanism behind fractional damping model parameters and their identification methods still need deeper theoretical support from materials science [3]. This results in poor model transferability and unclear physical meaning of parameters, which limits their predictive reliability in unverified operational conditions.

Second, the methodological level has the bottleneck of failed cross-scale association. There is no strong theoretical mapping between lumped-parameter models and continuum models, which results in a disconnection between parameter identification of zero-dimensional models and evolution prediction of high-dimensional fields [3]. As demonstrated by dynamic fracture analysis (Section 4.2) and equivalent SDOF control strategies (Section 4.3), quantitative associations between global temporal dynamics revealed by SDOF models and spatial stress fields, damage evolution, or dominant modes of continuum structures are yet to be established as an unresolved methodological bottleneck [9, 12]. This problem can only be solved by creating new mathematical tools that would allow establishing quantitative relationships between models at different scales.

Lastly, the system level presents the issue of disconnected technical components. The modeling, identification and control processes are usually considered as separate or sequential issues with no common optimization framework. This separation is reflected in particular terms as parameter identification which does not take into account control characteristics, physical models that do not consider the necessity to update in real time, and control designs on over-simplified models. As an example, a typical one is that the co-design of online parameter identification algorithms for fractional damping systems and their respective fractional-order controllers in real-time is still an open research problem[3, 4]. A deeper question is: can we create smart constitutive models whose structure or order can change dynamically according to the actual observation data, thus essentially circumventing the conventional path dependence of pre-defined model forms [4]?

5.3 Future Development Directions

Addressing these challenges, future research should seek breakthroughs in three key directions:

Developing Constitutive Theory Using Generalized Mechanical Principles: In the future, more work should be done to look into frameworks like fractional thermodynamics. The goal is to build nonlinear constitutive theories based on more fundamental physical laws that can consistently explain rate-dependent, memory-dependent, and non-local dissipation phenomena. This will give model parameters a clearer physical meaning [3, 7].

Creating Mathematical Techniques to Associate Multi-Scale Models: It is necessary to address the model order reduction theory and data-driven scale-bridging methods. This involves working on how to accurately project the dynamic information of high-dimensional continuum systems onto low-dimensional, even fractional-order, SDOF models using mathematical transformations, and providing spatial field validation for conclusions drawn from SDOF models in problems like dynamic fracture [9, 12].

Solving Real-Time Co-optimization Problems with Physical Constraints: In the case of the Intelligent Integration Paradigm, it is necessary to create embedded real-time simulation and physics-informed machine learning technologies. The aim is to reach online, fast calculation of complex dynamic models such as fractional models, and use this as the core to create algorithms that combine system identification, state estimation, and control decision-making within a single optimization framework, eventually forming a high-performance model-data-control closed loop [4].

6. Conclusion

This systematic review shows that the theoretical development of the single-degree-of-freedom spring-mass system is far from being a simple model improvement. It’s essentially a continuous process of tension adjustment based on the three central dimensions: “model accuracy”, “physical reality”, and “engineering applicability”. The linear theoretical system gives a full benchmark, but its value is best realized in the process of being surpassed. This paper, by breaking down nonlinear extensions into updates to the three fundamental physical dimensions (damping, stiffness, and inertia), clearly shows how physical mechanisms deepen and computational complexity rises with each model improvement. At present, the field is at an important point of shifting from separate nonlinear model research to building an integrated model-data-control system framework. When facing core problems such as the lack of a universal theory, failed cross-scale associations, and broken system loops, future research should combine first-principles modeling, advanced system identification, and intelligent control theory. That’s the key to solving these core challenges and advancing the field [4].

References

[1] Inman, D. J. (2013). Engineering vibration (4th ed.). Pearson. Print ISBN 9780273768449; eBook ISBN 9780273785217

[2] Rao, S. S. (2017). Mechanical vibrations (6th ed.). Pearson. ISBN 9780134361307.

[3] Nayfeh, A. H., & Mook, D. T. (1995). Nonlinear oscillations. Wiley-VCH. https://doi.org/10.1002/9783527617586

[4] Brunton, S. L., & Kutz, J. N. (2022). Data-driven science and engineering: Machine learning, dynamical systems, and control (2nd ed.). Cambridge University Press. https://doi.org/10.1017/9781009089517

[5] Podlubny, I. (1998). Fractional differential equations. Academic Press. Hardback ISBN 9780125588409; eBook ISBN 9780080531984

[6] Chen, K.-F., & Cai, C. (2017). Necessary and sufficient condition for eliminating static-deformation effects from the natural-frequency expression when the spring axis is tangent to the motion trajectory. College Physics, 36(11), 18–24. https://dxwl.bnu.edu.cn/CN/Y2017/V36/I11/18.

[7] Marino, L., & Cicirello, A. (2020). Experimental investigation of a single-degree-of-freedom system with Coulomb friction. Nonlinear Dynamics, Volume 99, pages 1781–1799. https://doi.org/10.1007/s11071-019-05443-2

[8] Kuttan, S. S., & Saha, A. (2024). Improving the Effectiveness of Frictional Damping by Varying the Normal Load in a System with One Degree of Freedom. In: Kumar, D., Sahoo, V., Mandal, A.K., Shukla, K.K. (eds) Advances in Applied Mechanics. INCAM 2022. Lecture Notes in Mechanical Engineering. Springer, Singapore. https://doi.org/10.1007/978-981-97-0472-9_3

[9] Kazarinov, N. A., Smirnov, A. A., & Petrov, Y. V. (2024). Revisiting mass-on-spring model to address key dynamic fracture effects. Theoretical and Applied Fracture Mechanics, 134, 104470. https://doi.org/10.1016/j.tafmec.2024.104470

[10] Smith, M. C. (2002). Synthesis of mechanical networks: The inerter. IEEE Transactions on Automatic Control, 47(10), 1648–1662. https://doi.org/10.1109/TAC.2002.803532

[11] Basili, M., De Angelis, M., & Pietrosanti, D. (2019). Defective two adjacent single degree of freedom systems linked by spring–dashpot–inerter for vibration control. Engineering Structures, 188, 480–492. https://doi.org/10.1016/j.engstruct.2019.03.030

[12] Antoulas, A. C. (2005). Approximation of large-scale dynamical systems. Society for Industrial and Applied Mathematics (SIAM). https://doi.org/10.1137/1.9780898718713

[13] Tian, Q. K. (2025). A new method for determining the optimal damping of vibration absorber. Noise and Vibration Control, 45(1), 269–275. https://doi.org/10.3969/j.issn.1006-1355.2025.01.042

[14] Zhong, Z., Cai, Z., & Qi, Y. (2022). Model-free adaptive control for single-degree-of-freedom magnetically levitated system. Journal of Southwest Jiaotong University, 57(3), 549–557. https://doi.org/10.3969/j.issn.0258-2724.20210624

[15] Li, P., Li, D., & Zhang, H. (2025). Dynamics study of a class of single degree of freedom double-sided constrained collision vibration systems. Journal of Dynamical Systems and Control, 14(1), 1–10. https://doi.org/10.12677/dsc.2025.141001

