Two-Client Scheduling with Order Selection

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Abstract: The paper addresses two-client scheduling with order selection on a machine. There are two clients *G* and *H*. Each having his own item sets. An item of each client needs to be selected before processing. Namely, an item is either selected to accept and process on the machine, or reject but require to pay a certain penalty for rejection. Decision makers want to minimize the sum of objections of accepted items and the rejection penalty of rejected items of client *G*. But at the same time, the sum of objections of accepted items and the rejection penalty of rejected items of client *H* can not exceed a constant. For the above problems, we first uncover crucial structural properties. Then analysis several different cases when items are processed on the machine. And develop two algorithms at last.

Keywords: Scheduling; Two-Client, Item; Acceptance; Algorithm

1. Introduction

Machine scheduling has wide practical applications. The aim is to present an assignment for items optimally to a single or a group of machines. But this assignment needs to meet some rules or constrains. Optimizing the schedules can give significant reduction of production costs, satify customer demand, and enhance production efficiency. With the development of market and complexity increase of production, manufacturers need to optimize all aspects of production process.

Multicriteria scheduling is one of the important branches of scheduling. There are two or more objections in this scheduling, and the target is to seek a sequence so that all objections are optimized or satisfied. Multi-client scheduling is a research topic in the multicriteria scheduling. In this model, we have $m \geq 2$ clients and each client has item sets $J_1, J_2, \cdots, J_m$,respectively. All the items share common machine resources. Moreover, a machine can only handle one item at each moment.

In the classical scheduling literatures, items are not allowed to select. All items must be processed on the production line. But in real world applications, it may not be true, due to the limited resources, the scheduler may need to select orders of clients. Thus, some items may be selected to accept and process on machines, some items may be selected to reject because of limited resources but need to pay a item-dependent rejection penalty. Bartal et al. [1] studied order selection firstly. Agnetis et al. [2] researched two clients scheduling without order selection. The following are papers which are related to our problems: Agnetis et.al. [3], He et.al. [4]. Chen and Li [5] study job shop problem with optional job rejection and propose an exact algorithm, a greedy algorithm is presented. Rault et.al [6] interest in two-client interfering item sets and propose genetic algorithms. Agnetis et.al. [7] analyze the structure of trade-off solutions for two-clients and propose an exact algorithm. Their algorithm is based on the Lagrangian relaxation of a MILP formulation for problem. Yu and Oron [8] study two-client problems on a parallel-batching machine. They show both their problems are strongly NP-hard. They motivate a constructive heuristic tailored to each problem. And address two metaheuristic algorithms, Simulated Annealing and Tabu Search. Use a comprehensive computational experiment to validate the efficiency of their algorithms.

Feng et al. [9] first studied two-client scheduling with order selection on a machine. Feng et al. [10] promote [9] to two uniform machines.

2. Problem Statement

This paper considers two-client problems with order selection on a machine. There are two clients G and H and each having his own item sets $J^G=\{J_1^G,\cdots,J_{n_G}^G\}$ and $J^H=\{J_1^H,\cdots,J_{n_H}^H\}$, respectively. We assume that $J^G\bigcap J^H=\varnothing$.Let $n=n_G+n_H$ denote the total number of items. Each item J_j^X has a processing time p_j^X and a due date d_j^X, $X\in\{G,H\}$. For each item, J_j^X is either accepted or rejected but need to pay a rejection penalty e_j^X, $X\in\{G,H\}$.Moreover, all items start to process at time zero. Let σ denote a feasible schedule of the n items. Write $P=\sum_{i=1}^{n_G}p_i^G+\sum_{i=1}^{n_H}p_i^H$. The completion time and lateness of item J_j^X, $X\in\{G,H\}$ in σ is denoted as $C_j^X(\sigma)$, $L_j^X(\sigma)=C_j^X(\sigma)-d_j^X$,respectively. Set $L_{\max}^X=\max\{L_j^X(\sigma)\}$ the maximum lateness of item J_j^X in σ. Let A_X and R_X be sets of accepted X - items and rejected X - items, respectively. $e(R_X)$ is the corresponding rejection penalty of items of clients X, E^X denotes total rejection penalty of all items of clients X. $X\in\{G,H\}$. The goal is to minimize $F_{\max}^G+e(R_G)$ subject to the constraint $F_{\max}^H+e(R_H)\le M$, where $F\in\{L_{\max},\sum C_j\}$,and M is a given positive integer. Two problems can be writen problem 1 $1\left|selection,L_{\max}^H+e(R_H)\le M\right|L_{\max}^G+e(R_G)$. and problem 2 $1\left|selection,\sum_{J_j^H\in A_H}C_j^H+e(R_H)\le M\right|\sum_{J_j^G\in A_G}C_j^G+e(R_G)$.

First, we assume that $M\le\min\{P,e(R_H)\}$. Otherwise, if $M>\min\{P,e(R_H)\}$, then all H - items can be rejected or processed after all G - items. Thus, the two-client problems are equivalent to the corresponding single-client problems.

3. Algorithm of Problem 1

In this section, we will give an algorithm for problem $1\left|selection,L_{\max}^H+e(R_H)\le M\right|L_{\max}^G+e(R_G)$.

Definition1. Earliest due date (edd) rule: items are processed in nondecreasing sequence of d_j^X, $X\in\{G,H\}$.

Theorem 1. Problem $1\left|selection,L_{\max}^H+e(R_H)\le M\right|L_{\max}^G+e(R_G)$, has an optimal sequence satisfying those accepted G - items and H - items are handled in edd order, respectively.

Proof. Using swap is easy to prove it.

By theorem 1, mark $J_1^G,J_2^G,\cdots,J_{n_G}^G$ and $J_1^H,J_2^H,\cdots,J_{n_H}^H$ such that $d_1^G\le d_2^G\le\cdots\le d_{n_G}^G$ and $d_1^H\le d_2^H\le\cdots\le d_{n_H}^H$,respectively.

Theorem 2. Problem $1\left|selection,L_{\max}^H+e(R_H)\le M\right|L_{\max}^G+e(R_G)$ can be solved in $O(n_Gn_HPE^GM^2)$ time.

Proof. Let $Y(i,j,t,E_i^G,E_j^H,L_B)$ be the minimum objective function value of client G satisfying:

(1) items that are considered in the current sequence are $J_1^G,J_2^G,\cdots,J_i^G$, $J_1^H,J_2^H,\cdots,J_j^H$.

(2) The maximum completion time of the items that are selected to accept among $J_1^G,J_2^G,\cdots,J_i^G$ and $J_1^H,J_2^H,\cdots,J_j^H$ is t.

(3) The total rejection penalty that are selected to be rejected among $J_1^G,J_2^G,\cdots,J_i^G$ is E_i^G.

(4) The total rejection penalty that are selected to be rejected among $J_1^H,J_2^H,\cdots,J_i^H$ is E_j^H.

(5) The current maximum lateness of the items that are selected to accept among $J_1^H,J_2^H,\cdots,J_j^H$ is L_B.

To guarantee the feasibility of H -jobs, we assume that $L_B + E_j^H \le M$.

In an optimal schedule for $J_1^G, J_2^G, \cdots, J_i^G$, $J_1^H, J_2^H, \cdots, J_j^H$. Thus have:

Case 1. J_i^G is selected to reject. So the makespan among $J_1^G, J_2^G, \cdots, J_{i-1}^G$ and $J_1^H, J_2^H, \cdots, J_j^H$ is t . Total rejection cost of clients G and H are $E_i^G - e_i^G$ and E_j^H , respectively. Therefore

$$Y\left(i, j, t, E_i^G, E_j^H, L_B\right) = Y\left(i-1, j, t, E_i^G - e_i^G, E_j^H, L_B\right) + e_i^G \quad (1)$$

Case 2. J_i^G is selected to be processed last. Then the makespan of the accepted items among $J_1^G, J_2^G, \cdots, J_{i-1}^G$ and $J_1^H, J_2^H, \cdots, J_j^H$ is $t - p_i^G$,

and the total rejection penalty of customer G and H are E_i^G and E_j^H , respectively. Then

$$Y\left(i, j, t, E_i^G, E_j^H, L_B\right) = E_i^G + \max\left\{Y\left(i-1, j, t - p_i^G, E_i^G, E_j^H, L_B\right) - E_i^G, t - d_i^G\right\} \quad (2)$$

Case 3. J_j^H is rejected. Then the maximum completion time of that are selected to accept items is t among $J_1^G, J_2^G, \cdots, J_i^G$ and $J_1^H, J_2^H, \cdots, J_{j-1}^H$. Total rejection penalty of clients G and H are E_i^G and $E_j^H - e_j^H$, respectively. The contribution of J_j^H to objective function is e_j^H . Thus

$$Y\left(i, j, t, E_i^G, E_j^H, L_B\right) = Y\left(i, j-1, t, E_i^G, E_j^H - e_j^H, L_B\right) + e_j^H \quad (3)$$

Case 4. J_j^H is selected to be processed last. Then the maximum completion time of the accepted items among $J_1^G, J_2^G, \cdots, J_i^G$ and $J_1^H, J_2^H, \cdots, J_{j-1}^H$ is $t - p_j^H$,

and the general rejection penalty of clients G and H are E_i^G and E_j^H , respectively. Moreover, $L_B = \max\left\{L_B', t - d_j^H\right\}$, where L_B' is the maximum lateness of the accepted H -items before J_j^H . Clearly, if $L_j^H = t - d_j^H < L_B$,then $L_B' = L_B$. and $0 \le L_B' \le L_B$, otherwise. Thus, in Case 4, we have

$$Y\left(i, j, t, E_i^G, E_j^H, L_B\right) = Y\left(i, j-1, t - p_j^H, E_i^G, E_j^H, L_B\right) \quad (4)$$

Summarize the above four cases and present algorithm A1 for problem $1\left|selection, L_{\max}^H + e\left(R_H\right) \le M\right|L_{\max}^G + e\left(R_G\right)$.

Algorithm A1

The initial conditions:

$$Y\left(0, 0, t, E_i^G, E_j^H, L_B\right) = \begin{cases} 0, t = E_i^G = E_j^H = L_B, \\ +\infty, \text{ otherwise.} \end{cases}$$

The recursive equation:

$$Y\left(i, j, t, E_i^G, E_j^H, L_B\right) = \begin{cases} Y\left(i-1, j, t, E_i^G - e_i^G, E_j^H, L_B\right) + e_i^G, \\ E_i^G + \max\left\{Y\left(i-1, j, t - p_i^G, E_i^G, E_j^H, L_B\right) - E_i^G, t - d_i^G\right\}, \\ Y\left(i, j-1, t, E_i^G, E_j^H - e_j^H, L_B\right) \\ Y\left(i, j-1, t, E_i^G, E_j^H - e_j^H, L_B\right), if L_j^H = t - d_j^H < L_B \\ \min\left\{Y\left(i, j-1, t - p_j^H, E_i^G, E_j^H, L_B'\right):\right\}, if L_j^H = t - d_j^H = L_B \end{cases} \quad (5)$$

The optimal value:

$$\min\left\{\begin{array}{l} Y\left(n_G, n_H, t, E_i^G, E_j^H, L_B\right): \\ 0 \le t \le P, 0 \le E_i^G \le E^G, 0 \le L_B + E_j^H \le M \end{array}\right\} \quad (6)$$

Algorithm A1 has at most $O\left(n_G n_H P E^G M^2\right)$ states. If $L_j^H = t - d_j^H = L_B$,then L_B is determined by t and the current scheduled item J_j^H . Therefore, there are at most $O\left(n_G n_H P E^G M\right)$ such states and each iteration needs $O\left(M\right)$ time or a constant time. Thus, the total running time is $O\left(n_G n_H P E^G M^2\right)$.

4. Algorithm of Problem 2

Definition 2. Shortest processing time (spt)

rule: items are processed in nondecreasing order of p_j^X, $X \in \{G,H\}$.

Theorem 2. Problem $1\left|selection, \sum_{J_j^H \in A_H} C_j^H + e(R_H) \le M\right| \sum_{J_j^G \in A_G} C_j^G + e(R_G),$ has an optimal sequence satisfying those accepted G - items and H - items are handled in spt order, respectively.

Proof. Using interchanging can prove it easily.

By theorem 2, mark $J_1^G, J_2^G, \cdots, J_{n_G}^G$ and $J_1^H, J_2^H, \cdots, J_{n_H}^H$ so that $p_1^G \le p_2^G \le \cdots \le p_{n_G}^G$ and $p_1^H \le p_2^H \le \cdots \le p_{n_H}^H$, respectively.

Theorem 3. Problem $1\left|selection, \sum_{J_j^H \in A_H} C_j^H + e(R_H) \le M\right| \sum_{J_j^G \in A_G} C_j^G + e(R_G),$ can be solved in $O(n_G n_H P M^2)$ time.

Proof: Let $Z(i,j,t,E_j^H,q)$ be the minimum objective function value of client G satisfying:

(1) items in consideration are $J_1^G, J_2^G, \cdots, J_i^G$, $J_1^H, J_2^H, \cdots, J_j^H$ in current sequence.

(2) The maximum completion time of the items that are selected to accept among $J_1^G, J_2^G, \cdots, J_i^G$ and $J_1^H, J_2^H, \cdots, J_j^H$ is t.

(3) The general rejection penalty that are selected to be rejected among $J_1^H, J_2^H, \cdots, J_i^H$ is E_j^H.

(4) The general completion time of the accepted H items among $J_1^H, J_2^H, \cdots, J_j^H$ is q, where $q + E_j^H \le M$.

Then we have:

Case 1. J_i^G is selected to reject. Thus the maximum completion time of the selected to processed items is t among $J_1^G, J_2^G, \cdots, J_{i-1}^G$ and $J_1^H, J_2^H, \cdots, J_j^H$. Total rejection penalty of client H is E_j^H. The contribution of J_i^G to objective function is e_i^G. Thus

$$Z(i,j,t,E_j^H,q) = Z(i-1,j,t,E_j^H,q) + e_i^G. \quad (7)$$

Case 2. J_i^G is selected to be processed last. The makespan of the accepted items among $J_1^G, J_2^G, \cdots, J_{i-1}^G$ and $J_1^H, J_2^H, \cdots, J_j^H$ is $t - p_i^G$, and the general rejection penalty of client H is E_j^H. Then

$$Z(i,j,t,E_j^H,q) = Z(i-1,j,t-p_i^G,E_j^H,q) + t. \quad (8)$$

Case 3. J_j^H is rejected. Then the maximum completion time of that are selected to accept items is t. Total rejection penalty of client H is $E_j^H - e_j^H$. Thus

$$Z(i,j,t,E_j^H,q) = Z(i,j-1,t,E_j^H - e_j^H,q). \quad (9)$$

Case 4. J_j^H is selected to be processed last. So the makespan of the accepted items among $J_1^G, J_2^G, \cdots, J_i^G$ and $J_1^H, J_2^H, \cdots, J_{j-1}^H$ is $t - p_j^H$, and the general rejection penalty of client H is E_j^H. The total completion time of the accepted H - items among $J_1^H, J_2^H, \cdots, J_{j-1}^H$ is $q - t$. Therefore,

$$Z(i,j,t,E_j^H,q) = Z(i,j-1,t-p_j^H,E_j^H,q-t). \quad (10)$$

Summarize the above four cases and have the following algorithm A2 for $1\left|selection, \sum_{J_j^H \in A_H} C_j^H + e(R_H) \le M\right| \sum_{J_j^G \in A_G} C_j^G + e(R_G),$ problem

Algorithm A2

The initial conditions:

$$Z(0,0,t,E_j^H,q) = \begin{cases} 0, t = E_j^H = q = 0, \\ +\infty, \text{ otherwise.} \end{cases}$$

and

$Z(i,j,t,E_j^H,q) = +\infty$, if $E_j^H + q > M$.

The recursive equation:

$$Z(i,j,t,E_j^H,q) = \begin{cases} Z(i-1,j,t,E_j^H,q) + e_i^G, \\ Z(i-1,j,t-p_i^G,E_j^H,q) + t, \\ Z(i,j-1,t,E_j^H - e_j^H,q), if E_j^H + q < M, \\ Z(i-1,j,t-p_j^H,E_j^H,q-t), if E_j^H + q < M. \end{cases} \quad (11)$$

The optimal value:

$$\min\left\{\begin{aligned}&Z\left(n_G,n_H,t,E_j^H,q\right):\\&0\le t\le P,0\le E_j^H\le E^H,0\le E_j^H+q\le M\end{aligned}\right\}. \quad (12)$$

Algorithm2 has at most $O\left(n_G n_H PM^2\right)$ states. Each iteration needs $O(1)$. So the computational complexity is $O\left(n_G n_H PM^2\right)$.

5. Conclusions

The paper studies problems $1\left|selection, L_{\max}^H + e\left(R_H\right) \le M\right|L_{\max}^G + e\left(R_G\right).$ and $1\left|selection, \sum_{J_j^H\in A_H} C_j^H + e\left(R_H\right) \le M\right|\sum_{J_j^G\in A_G} C_j^G + e\left(R_G\right).$. For the two problems, as a first step, we analysis structural properties. Then give several different cases when items are processed on the machine. And develop two algorithms at last. For the further issues, one can address interfering job sets or approximation algorithms or other machine environment.

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