

The Mineral Resources Pricing Model Based on the Mix-Copula-EGARCH Model

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Abstract: Effective assessment of mineral resources has an important impact on the market allocation of mineral resources. The traditional discounted cash flow (DCF) method fails to appropriately express the uncertainty information about the market. In this paper, the pricing of mineral assets is studied from the perspective of real options. Firstly, the mining resource mining rights are expressed as binary real options with the underlying assets taken as the resource reserves value and the convenience yield. Next, the calculation model of the convenience yield is given. Considering the asymmetric volatility process of assets, the EGARCH model is selected to represent the volatility of assets, and the marginal distribution information of assets income under the risk-neutral measure is obtained through the transformation of the risk-neutral measure. Considering the impact of the complex dynamic relationship among assets on mineral resources pricing, the Gumbel Copula function and the Clayton Copula function are selected to describe the upper-tail and lower-tail correlation of assets, while the Mix-Copula function combining these two Copula functions is established to describe the non-linear correlation of assets. By combining the advantages of both the EGARCH model and the Mix-Copula function, the mineral resource pricing model based on the Mix-Copula-EGARCH model is established to price mineral resources. The research objects of four non-ferrous metals, i.e. zinc, nickel, copper and tin, are selected, the Monte Carlo method is adopted for empirical analysis of them, and comparisons with traditional resource pricing methods are also made. The empirical results show that the mineral resource pricing model based on the Mix-Copula-EGARCH model proposed in this paper can better reflect the market value of mineral resources and it is superior to other traditional pricing methods.

Keywords: Mineral Resources; EGARCH Model; Copula Function; Real Options; Monte Carlo

1. Introduction

Along with the rapid economic growth, the demands for resources are expanding. Establishing an effective market assessment mechanism and fully reflecting the value of mineral resources serve as the key to the development of resource market. However, the current resource market pricing mechanism remains imperfect, which causes the market price of mineral resources be excessively low, prevents the full reflection on the market value of mineral resources, and affects the market efficiency in the allocation of mineral resources. In order to make reasonable market pricing of mineral resources, it is prerequisite to establish a scientific effective method to evaluate the value of mineral resources. At present, mineral resources are mainly evaluated by the discounted cash flow (DCF) method. The assessment principle of this method is to select an appropriate discount rate to discount the future expected cash flow. The DCF method is widely used in practice because it is simple and practical. However, this static assessment method often needs to assume that the market situation in the future remains basically unchanged. As a result, the market uncertainty faced in the mining process is ignored, leading to inaccurate valuation. For a long time, scholars have applied the real option theory to the assessment of mineral resources. Brennan and Schwartz (1985)^[1] have used the yield spread generated between commodity futures and spot prices to evaluate the option value of mineral mining. Paddock, Siegel and Swith (1988)^[2] have discussed the assessment of coastal oil deposits, established an American option pricing model, compared it with traditional methods, and pointed out that the more uncertainties in the future, the greater the value of real options.

Gibson and Schwartz (1989) ^[3] have considered the resource valuation method based on the convenience yield and constructed a crude oil contract pricing model through random differential equation. Considine and Timothy (1997) ^[4] have pointed out that continuous innovations in futures, options and derivatives allow flexible trading, speculation and hedging of assets, and these financial products provide useful market information for the assessment of mineral resources. Insley and Wirjanto (2010) ^[5] have used the risk-neutral pricing method and the cash flow method to empirically analyze the value of natural resources. The empirical results show that the risk-neutral pricing method is better than the cash flow method. Paschke and Prokopczuk (2010) ^[6] have used the Autoregressive Integrated Moving Average (ARIMA) model to estimate the dynamic change process of prices, evaluated the real options of resources based on this result, and got better market assessment results. Fan and Zhu (2010) ^[7] have incorporated investment environmental factors into the value assessment of petroleum investment, applied real option technology to construct a multinational petroleum investment assessment model, and also applied this model to empirically analyze the value of China's transnational oil investment projects. Zhu and Fan (2011) ^[8] have constructed a CCS (Carbon Capture and Storage) investment value assessment model by utilizing the real option theory, which has provided effective technical support for the assessment of carbon investment value. Guj P (2011) ^[9] has introduced a method of real option assessment and illustrated how a typical "in/out" protocol, as an initial condition for establishing joint venture (JV), is represented as a series of consecutive compound real options. Dehghani H (2013) ^[10] believes that the value of mine resources may be affected by a variety of market uncertainties, while traditional assessment methods, such as the discounted cash flow (DCF) method for mining projects, can not represent the true value of mineral projects. He has used the DCF method and the ROV method to calculate the net present value (NPV) of mineral resources under the assumption that operating costs and metal prices are uncertain. The calculation results show that NPV calculated by ROV is 40% greater than DCF. Haque M A (2014) ^[11] has used hedging strategy and commodity futures contract based on the geometric Brownian motion model to derive

Black-Scholes equation partial differential equation (PDE). By using the finite difference method (FDM) and MatLab software, the numerical solution of PDE is given. By taking a gold mine as an example, PDE is numerically solved by adopting the real option valuation method and giving considerations to the alternatives of mining projects. Kesler S E, Simon A C and Simon A F (2015) ^[12] have studied the important role of mineral resources in economic development from multiple perspectives of geology, environment and economy. Considering that the traditional discount cash flow (DCF) method can not solve the problem of market uncertainty and operational flexibility in mineral resource assessment, Zhang K, Nieto A and Kleit AN (2015) ^[13] have estimated the mine's in-situ value by determining the optimal price threshold of mining activities, combined with the real option value (ROV). The results show that the ROV method provides a realizable easy-to-operate solution for in-situ value assessment of mines and it serves as an effective strategy for mineral resources assessment. Zhang K and Kleit A N (2016) ^[14] have established a theoretical two-stage economic model to derive the value of mineral resource inventory and its impact on the optimal recovery ratio. Through the derivation and analysis of optimal conditions of objective profit functions, it is found that mineral inventory options can significantly improve the profits of mines. Sergeev I B and Lebedeva O Y (2016) ^[15] have studied how mineral reserves and resources are reflected and assessed in financial statements of mining companies, and how these mineral assets affect the market values of these companies. The conclusion shows that ore reserves and resources are not clearly recognized in financial reports, which leads to the difference between the market values and the book values of these companies. In this regard, this point should be fully taken into account at mergers and acquisitions. Savolainen J, Collan M and Luukka P (2017) ^[16] have proposed a detailed system dynamic (SD) model of metal mining investment, which can be used for prior earning capacity analysis and management & operation analysis. It shows how to use the SD model to analyze the profit effects of three kinds of operating real options. By taking three different future metal price scenarios and two different random processes as examples, the importance of correct process

selection for future metal price path modeling is illustrated. Haque MA, Topal E and Lilford E (2017) [17] have considered the instability of cash flows generated by mining projects and the vulnerability to exogenous variables such as commodity prices and exchange rates, and assessed mineral resources by integrating commodity prices and their volatility rates, exchange rates and their volatility rates, as well as related parameters among them into the real option model. It is revealed in research findings that if only the uncertainty of commodity prices is taken into account when assessing the value of a project, without considering the joint effect of the uncertainty of commodity prices and exchange rates, the project value may be overestimated.

Through the analysis of the literatures on real options, it can be found that real options have become a powerful tool for pricing mineral resources along with the gradual improvement of real options pricing theory. However, it is usually assumed in current researches that the volatility ratio of the underlying assets remains unchanged and the relationship among the underlying assets is linear. In fact, all these are far from the reality. In this paper, the pricing of mineral assets is studied from the perspective of real options. The mining resources mining rights are expressed as binary real options with the underlying assets taken as the resource reserves value and the convenience yield. Firstly, the calculation model of the convenience yield is established. Then, considering the impact of fluctuations of spot assets and the non-linear correlation among assets on mineral resources pricing, the EGARCH volatility model is selected to represent the fluctuation process of spot assets. By using the equivalent martingale measure method, the marginal distribution of return on assets is obtained through the transformation of the risk-neutral measure. By respectively utilizing the Gumbel Copula function and the Clayton Copula function to describe the upper-tail and lower-tail correlation of assets, the Mix-Copula function combining these two kinds of Copula functions is established to describe the complex dynamic correlation of assets. By combining the advantages of the EGARCH model and the Mix-Copula function, the Mix-Copula-EGARCH model based on the convenience yield and the spot price is established to set prices for mineral resources.

The research objects of four non-ferrous metals, i.e. zinc, nickel, copper and tin are selected, and the Monte Carlo method is also adopted for empirical analysis of them, hoping to provide a reference for the value assessment of mineral resources.

The structure of the remaining parts of this paper is as below: the pricing model of mineral resources is given in Part 2, the empirical analysis is provided in Part 3, and the conclusion is given in Part 4.

2. The Pricing Model of Mineral Resources

2.1 the Discount Cash Flow Method for Mineral Resources

According to the theory of microeconomics, the main factors affecting the prices of mineral resources include the recoverable reserve q , the product price P , the operating cost C , the taxes and fees G , etc. The net cash flow of mineral resources mining projects that can be obtained at the moment t is as below:

$$S(t) = q(t)(P(t) - C'(t))(1 - G(t)) \quad (1)$$

According to the traditional discount method for net cash flow, the market price of a mineral resources mining project can be obtained as below:

$$DCF = \sum_{t=1}^T \frac{(S(t) - I(t))}{(1 + R)^t} \quad (2)$$

Where, R stands for the discount rate and $I(t)$ for the mining cost invested during different periods. Similarly, according to the relevant theory of real options, mineral resources and call options have corresponding characteristics, and their values can also be regarded as European call options with the underlying assets defined as the net cash flow $S(t)$, execution price as the outlay cost $I(t)$ required by the project, and maturity date T as the mining period. Next, mineral resources are analyzed with the real option method.

2.2 the Convenience Yield of Mineral Resources

The mineral resources mining rights can be considered as binary real options with the underlying assets taken as the resource reserves value and the convenience yield, and the actual value of this real option is jointly determined by

the joint distribution of assets and the convenience yield. Domestic and foreign scholars have conducted in-depth researches on the dynamic relationship between commodity futures and spot, believing that the correct understanding of the convenience yield plays an important role in the pricing of mineral resources. Brennan (1985)^[18] has defined the convenience yield as value-added gains naturally generated by physical commodity. Such value-added gains can only be obtained by physical commodity holders, while holders of asset-derived contracts (such as financial options) do not. Gibson and Schwartz (1989)^[19] have revised the assumption that the convenience yield is fixed when evaluating petroleum derivative contracts, and adopted the random differential equation method to estimate the convenience yield. However, it is very difficult to obtain an analytical solution through the application of random differential equation in the estimation of the convenience yield. Moreover, the realization is also relatively difficult. In this paper, another method will be employed to estimate the convenience yield and the volatility of convenience yield by utilizing the asset volatility model. Under the no-arbitrage condition, assuming that only risk-free compensation can be obtained for all assets, the convenience yield essentially becomes "dividend" obtained by spot holders of spot mineral resources, and the futures price is satisfied for the underlying assets:

$$F_t^T = P_t e^{\sum_{i=t+1}^T (r_f - D_i)} \tag{3}$$

Where, F_t^T is the price of the futures at the maturity date T , P_t and D_t are the spot price and the convenience yield at the time t , and r_f stands for the risk-free rate. To simplify the formula (3), the formula for convenience yield can be obtained:

$$\sum_{i=t+1}^T (r_f - D_i) = \ln(F_t^T) - \ln(P_t) \tag{4}$$

$$\sum_{i=t}^T (r_f - D_i) = \ln(F_{t-1}^T) - \ln(P_{t-1}) \tag{5}$$

$$D_t = r_f + (\ln(F_t^T) - \ln(F_{t-1}^T)) - (\ln(P_t) - \ln(P_{t-1})) = r_f + r_t^F - r_t^S \tag{6}$$

Where, r_t^F and r_t^S respectively stand for the logarithmic yield of futures prices and spot

prices during the period from $t-1$ to t . Formula (6) shows that the convenience yield is closely related to futures prices and spot prices. According to formula (6), we can estimate the convenience yield based on the prices of spot market and futures market.

2.3 the Real Option Pricing Model of Mineral Resources

Resource mining rights have the same option characteristics as call options. When investing in mineral exploitation, investors need to assess whether the received value of mineral resources is greater than the invested cost. If such condition is satisfied, investors are willing to obtain mineral exploitation rights. Otherwise, investors may give up the right to own mineral exploitation. Therefore, the value of the mineral resources mining rights should satisfy the following at the mining period T :

$$V_T = \max\{S_T - K, 0\} = \max\{P_T \cdot q - K, 0\} \tag{7}$$

Where, V_T stands for the value of mining rights at the maturity date, S_T for the reserves value of mineral resources at the moment T , K for the expected revenue of the investment cost, P_T for the unit price of resources at the moment T and q for the reserves of resources to be exploited. According to the principle of risk neutrality, under the condition of no arbitrage in the market, there must be an equivalent martingale measure Q , so that the value of mineral resources mining rights can be satisfied at the moment T under this measure:

$$V_T = e^{-r_f(T-t)} E^Q(V_T) = e^{-r_f(T-t)} E^Q(\max\{P_T \cdot q - K, 0\}) \tag{8}$$

Where, P_T meets:

$$P_T = P_t e^{\sum_{i=t+1}^T r_i^S} \tag{9}$$

According to the above formulas (6) -(9), it can be concluded as below:

$$V_t = e^{-r_f(T-t)} E^Q(\max(q \cdot S_t e^{\sum_{i=t+1}^T r_i^S} - K, 0)) = e^{-r_f(T-t)} E^Q(\max(q \cdot S_t e^{\sum_{i=t+1}^T (r_f + r_i^F - D_i)} - K, 0)) \tag{10}$$

As indicated in formula (10), the mineral resources mining rights can be expressed as binary real options with the underlying assets taken as the resource reserves value and the

convenience yield. The key to calculate the mineral resources mining rights is to determine the joint probability distribution and convenience yield of the underlying assets under the risk neutrality measure Q . Unlike financial options, which can be traded freely in financial markets, real options are often not traded in a timely manner. In this way, mining rights have the characteristics of European options in terms of implementation. Thus, the remaining mining quantity can be divided into n parts, and these mining rights will be executed year by year during the remaining mining period. The total value of the mineral resources mining rights can thus be obtained as:

$$V = \sum_{T=1}^n V_t \tag{11}$$

Where, n stands for the effective period of the mineral resources mining rights.

2.4 the Conversion of Asset Fluctuation Process under Risk Neutral Conditions

For accurate pricing of the mineral resources, in addition to determining the convenience yield of mineral resources, there is also need to determine the marginal distribution of the information value under the condition of risk-neutral measure Q . The distribution information of assets can be obtained through the residual error filtered by asset volatility equation, and the selection of an effective volatility model can more accurately express the distribution information of asset margins. Engle (1982) [20] has proposed the famous ARCH model in which the conditional variance is applied for the estimation of asset volatility. Bollerslev (1986) [21] has proposed the general autoregressive conditional heteroskedasticity (GARCH) model in which fewer lag orders are possessed if compared with the ARCH model and fewer parameters can be adopted to express the persistence of volatility. This GARCH model is as below:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1)$$

$$\sigma_t^2 = a_t + \sum_{i=1}^q a_i \sigma_{t-i}^2 + \sum_{i=1}^p \beta_i \varepsilon_{t-i}^2 \tag{12}$$

In order to analyze the phenomenon of volatility smiles in the option pricing model, Duan (1995) [22] has used the GARCH model to represent the volatility of assets, introduced the GARCH

model into the option pricing model, provided a kind of method for transforming the distribution of the underlying assets under risk-neutral conditions, and allowed assets to be subject to the GARCH process. Duan has pointed out that under the condition of satisfying the local risk-neutral value relationship (LRNVR), the GARCH model of the yield rate r_t under the risk-neutral measure Q can be obtained by changing the drift term μ_t , that is,

$$r_t = r_f - \frac{\sigma_t^2}{2} + \varepsilon_t^*, \quad \varepsilon_t^* \sim N(0, \sigma_t)$$

$$\sigma_t^2 = a_t + \sum_{i=1}^q a_i \sigma_{t-i}^2 + \sum_{i=1}^p \beta_i (r_{t-i} - \mu_{t-i})^2 \tag{13}$$

Where, the mean value of ε_t^* under the risk-neutral measure Q is 0, the variance is the

normal distribution of σ_t , and r_f stands for risk-free. The assets price is characterized by volatility aggregation, leverage effect and so on. However, the GARCH model is can not be used to distinguish the impacts of positive and negative jitter on fluctuations. In reality, the reactions of the assets price to positive jitter (good news) and negative jitter (negative news) are different. Many empirical tests show that the period of higher volatility is often closely associated with the decline of stock market, while the period of lower volatility is often closely related to the rise of stock market. In order to allow the asymmetric effect to be manifested when assets face positive and negative impacts in the GARCH model, Nelson (1991)[23] has introduced the asymmetric characteristics of fluctuation into the GARCH model, and proposed the EGARCH model to reflect the asymmetric effect of the yield rate. This EGARCH model is as below:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1)$$

$$\ln \sigma_t^2 = a_0 + \sum_{i=1}^p a_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^q \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^r \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \tag{14}$$

In the EGARCH model, the asymmetric effects of market price impact on asset fluctuations is considered, while the persistence of fluctuations can also be effectively described. According to Duan's method, under the condition of satisfying the local risk neutral value relationship (LRNVR), the EGARCH model of the yield rate r_t under the risk-neutral measure Q can be obtained by changing the drift term μ_t , namely:

$$r_t = r_f - \frac{\sigma_t^2}{2} + \varepsilon_t^*, \quad \varepsilon_t^* \sim N(0, \sigma_t)$$

$$\ln \sigma_t^2 = a_0 + \sum_{i=1}^p a_i \left| \frac{r_{t-i} - \mu_{t-i}}{\sigma_{t-i}} - E\left(\frac{r_{t-i} - \mu_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^q \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^r \gamma_k \frac{r_{t-i} - \mu_{t-i}}{\sigma_{t-k}} \quad (15)$$

Where, r_f stands for risk-free. The existence of leverage effect can be verified by the hypothesis of $\gamma_k < 0$. Compared with the GARCH model, the EGARCH model is more accurate in describing the asset volatility, and more flexible and convenient in the solving process. Through the transformation of the risk-neutral measure, the distribution of the yield rate under the risk-neutral measure Q can be obtained, which provides a theoretical basis for the pricing of mineral resources.

2.5 the Joint Distribution of Mineral Resources

For the pricing of the mineral resources, in addition to the risk neutral martingale pricing, there is also need to know the joint distribution of the underlying assets under the risk neutral condition. Sklar (1959)^[24] has proposed the Copula theory. Called as the "joint function" that connects the joint distribution function of multidimensional random variables with its marginal distribution, Copula functions have advantages in describing the joint distribution of multiple random variables. Many researchers have used Copula functions to describe the non-linear correlation structure among assets, and established the option pricing model of multi-financial assets. The theorem of Copula functions is as below: for any joint distribution

F and its marginal distributions F_1 and F_2 , there must be a Copula function C that satisfies the following:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \quad (16)$$

If F_1 and F_2 are continuous, then C is uniquely determined. Moreover, if F_1 and F_2 are marginal distributions of random variables and C is a Copula function, then the function F will be the joint distribution function of F_1 and F_2 obtained by formula (16). As pointed out by Patton (2008)^[25], if distributions under the risk-neutral measure exist in F_1 and F_2 , $C(F_1(x_1), F_2(x_2))$ will be the joint distribution

under risk-neutral conditions. At present, there are a lot of Copula functions in research literatures, and the commonly-used binary Copula functions include Gauss Copula, T-Copula, Gumble Copula and Clayton Copula. In these four binary Copula functions, considering the advantages of Gumble Copula function and Clayton Copula function in describing the upper-tail correlation and lower-tail correlation of assets, a mix Copula function combining these two kinds of Copula functions is established to describe the complex dynamic correlation of assets. The structure of these two Copula functions is as below:

(1) Gumbel Copula Function

When the generating function φ of Archimedes Copula function C satisfies $\varphi(u) = (-\ln(u))^\theta$, where, $\theta > 1$, the joint distribution function of the Gumbel Copula function should be as below:

$$C_{Gumbel}(u_1, \dots, u_N) = \exp\left\{-\left[\sum_{i=1}^N (-\ln u_i)^\theta\right]^{1/\theta}\right\}, \theta > 1 \quad (17)$$

(2) Clayton Copula Function

When the generating function φ of Archimedes Copula function C satisfies $\varphi(u) = u^{-\theta} - 1$, where, $\theta > 1$, the joint distribution function of the Clayton Copula function should be as below:

$$C_{Clayton}(u_1, \dots, u_N) = \left[\sum_{i=1}^N u_i^{-\theta} - N + 1\right]^{-1/\theta}, \theta > 0 \quad (18)$$

By integrating the Gumbel Copula function and the Clayton Copula function, the Mix-Copula model can be established as below:

$$C_{mix} = \varpi_1 C_{Gumbel} + \varpi_2 C_{Clayton} \quad (19)$$

Where, C_{mix} represents the mixed Copula function consisting of the linear combination of two Copula functions. C_{Gumbel} and $C_{Clayton}$ represent the Gumbel Copula function and the Clayton Copula function, respectively. $\varpi_i (i = 1, 2)$ represents the extent to which each Copula function affects the Mix-Copula function, and $\sum_{i=1}^n \varpi_i = 1$ ($0 \leq \varpi_i \leq 1$). The Mix-Copula function is associated with each Copula function by the weight coefficient ϖ_i , and the correlation

between assets is represented by different coefficients θ in each Copula function. In this way, the shortcomings of using a single Copula function to describe the correlation between assets can be avoided.

2.6 the Mineral Resource Pricing Model Based on the Mix-Copula-EGARCH Model

Based on the real option theory, combining the advantages of the EGARCH model and the Mix-Copula function, this paper establishes a Mix-Copula-EGARCH model based on the convenience yield and the spot price to price mineral resources by utilizing the equivalent martingale measure method. Firstly, the convenience value of mineral resources can be calculated by formula (6), and then the EGARCH model is selected to represent the fluctuation process of assets. Under the condition of the risk-neutral measure Q , the marginal distribution of the yield rate under the risk-neutral measure Q can be obtained by the transformation of the risk-neutral measure. When describing the upper-tail and the lower-tail correlation of assets, the Gumbel Copula function and the Clayton Copula function are respectively used to establish the Mix-Copula function to describe the complex dynamic correlation of assets and the joint distribution of assets. The above market price variables are substituted into formulas (10) and (11) to get the mineral resources pricing model based on the Mix-Copula-EGARCH model. The specific estimation methods of the mineral resources pricing model based on the Mix-Copula-EGARCH model are as below:

- (1) The convenience value of mineral resources is calculated according to formula (6) in Part 2.2.
- (2) According to the method introduced in Part 2.4, EGARCH parameters and marginal distribution parameters of the underlying assets under the risk-neutral measure Q are estimated.
- (3) The Mix-Copula function is used to estimate the related structure of the underlying assets, the MLE (Maximum Likelihood Estimate) method is used to estimate relevant parameters, and the joint distribution of the underlying assets under the risk-neutral measure is obtained according to the Copula theory in Part 2.5.
- (4) The value of the mineral resources is calculated by formulas (10) and (11) in Part 2.3.

Since the structure of the Mix-Copula-EGARCH model is relatively complex, it is impossible to give an analytical solution. Thus, the Monte Carlo method is applied in this paper to analyze the mineral resources pricing, so as to get the specific value of mineral resources.

3. Empirical Analysis

3.1 Empirical Objects and Data Processing

In the empirical part of this paper, the analysis focuses on the effects of the traditional DCF pricing method, the B-S option pricing method based on fixed volatility, and the mineral resources pricing model based on the Mix-Copula-EGARCH model. The market trading prices for four non-ferrous metals, i.e. zinc, nickel, copper and tin, from January 1, 2015 to December 30, 2016 are selected for estimation. The data are sourced from RESSET/DB database. Assuming that the risk-free annual interest rate is 4%, and the social average interest rate of mining investment is 8%, will be the discount rate in the DCF method.

3.2 the Mineral Resources Option Value Estimation Based on the Monte Carlo Method

In view of the excessive complexity of the mineral resource pricing model based on the Mix-Copula-EGARCH model proposed in this paper and the absence of an analytical solution, the MLE method is used to analyze the parameters of the EGARCH model and the Mix-Copula model at first in this paper, and then the obtained result is substituted into the model, and the Monte Carlo method is used to estimate the mineral resources pricing, so as to obtain the specific value of the real option. In this paper, the cash flow at the beginning of the period is assumed to be $S_0 = 1$. The mining period of resources is 10 years, divided into 5 mining stages, and each stage is 2 years. The investment cost I of each stage remains constant, and the mining rights are executed year by year. By pricing the mining rights of different holding periods separately, and then adding up, the total real option value of the mineral resources can be obtained. Specific steps are as below:

- (1) Estimate the parameters of the EGARCH model under risk-neutral conditions, and estimate the nonlinear correlation between assets through the Mix-Copula function;

- (2) Generate a price path for the underlying assets (spot price and convenience yield) under risk-neutral conditions according to the outcomes of (1);
- (3) Generate 10,000 price paths for the underlying assets under risk-neutral conditions by repeating the process (2) for 10,000 times.
- (4) Calculate the average value of the maturity price of the underlying assets in process (3), and obtain the current value of the mineral resources through the discount of the risk-free interest rate.

3.3 Comparisons of Different Mineral Resource Pricing Models

In order to prove the validity of the mineral resources pricing model based on the Mix-Copula-EGARCH model proposed in this paper, corresponding traditional models are selected for comparisons, and the relevant models are as below:

- (1) the DCF model (the traditional discounted cash flow method, mainly based on formula (2);
 - (2) the B-S model (underlying assets are with normal distribution, and historical volatility replaces future volatility);
 - (3) the Mix-Copula-EGARCH model (the mineral resources pricing model based on the Mix-Copula-EGARCH model);
- Firstly, the pricing differences among different resource pricing models under different mining costs are analyzed. Pricing results of the four mineral resources (Zinc, Nickel, Copper and Tin) under different input costs are provided in Table 1 and Table 4 by use of the DCF model, the B-S model and the Mix-Copula-EGARCH model, respectively.

Table 1. Input Cost I (Execution Price) and Resource Value of Nickel Resource

Input Cost I	1.00	1.20	1.40	1.60	1.80	2.00
DCF option value	0.64	1.10	1.24	1.39	1.54	1.74
B-S option value	0.92	1.27	1.39	1.43	1.91	2.15
Mix-Copula-EGARCH model option value	1.31	1.46	1.51	1.87	2.21	2.38

From Table 1 to Table 4, it can be observed that the resource value obtained by the DCF method is less than the option value given by the B-S model and the Mix-Copula-EGARCH model under different mining cost conditions, and in some cases (such as higher mining costs), the value of resource is negative. This shows that the decision-making value in mining process is neglected when using the DCF method to estimate the value of resources, which can easily lead to decision-making errors. Meanwhile, it

can also be observed that the resource value given on the basis of the Mix-Copula-EGARCH model is higher than that of the B-S model under different

Table 2. Input Cost I (Execution Price) and Resource Value of Copper Resource

Input Cost I	1.00	1.20	1.40	1.60	1.80	2.00
DCF option value	0.51	0.69	0.85	0.94	1.42	1.67
B-S option value	0.73	0.98	1.24	1.37	1.88	1.96
Mix-Copula-EGARCH model option value	0.98	1.17	1.55	1.56	2.06	2.21

Table 4. Input Cost I (Execution Price) and Resource Value of Tin Resource

Input Cost I	1.00	1.20	1.40	1.60	1.80	2.00
DCF option value	0.84	0.94	1.20	1.53	1.53	1.84
B-S option value	1.23	1.34	1.39	1.74	1.62	2.13
Mix-Copula-EGARCH model option value	1.34	1.47	1.67	1.78	1.98	2.46

mining cost conditions, showing that the sensitivity of resource value to volatility is relatively high and the selection of an effective asset volatility model can improve the accuracy of mineral resource value pricing.

Secondly, the pricing differences among different resource pricing models under different mining periods are analyzed. Pricing results of the four mineral resources (Zinc, Nickel, Copper and Tin) under different input costs are provided from Table 5 and Table 8 by use of the DCF model, the B-S model and the Mix-Copula-EGARCH model, respectively.

Table 5. Mining Period and Resource Value of Zinc Resource

Mining Period	2	4	6	8	10
The DCF option value	1.31	2.45	3.46	4.55	5.89
The B-S option value	1.47	2.79	3.21	4.68	6.31
The Mix-Copula-EGARCH model option value	2.49	5.23	7.24	9.62	11.71

Table 6. Mining Period and Resource Value of Nickel Resource

Mining Period	2	4	6	8	10
The DCF option value	1.29	2.45	3.53	4.91	6.20
The B-S option value	2.26	5.38	8.08	10.12	13.72
The Mix-Copula-EGARCH model option value	2.67	5.42	8.15	12.37	14.93

Table 7. Mining Period and Resource Value of Copper Resource

Mining Period	2	4	6	8	10
The DCF option value	0.96	1.85	2.94	4.21	5.17
The B-S option value	1.43	2.64	4.22	5.09	6.48
The Mix-Copula-EGARCH model option value	1.79	3.19	4.77	6.52	7.97

In Figures 5 to Table 8, it is shown that the values of the four mineral resources (zinc, nickel, copper and tin) grow along with the increase of the mining period (increase in mineral reserves). The resource prices respectively given by the DCF model and the Mix-Copula-EGARCH model are the lowest and the highest, while the resource price given by the B-S model falls in between. This shows that the decision value of the manager can be effectively considered in the Mix-Copula-EGARCH model, and the adoption of a more efficient volatility model in estimating the future risk of resource exploitation combines more market price information about mineral

Table 8. Mining Period and Resource Value of Zinc Resource

Mining Period	2	4	6	8	10
The DCF option value	1.53	2.61	4.35	5.79	7.24
The B-S option value	2.42	4.96	7.23	9.61	12.30
The Mix-Copula-EGARCH model option value	2.57	5.17	7.36	10.23	13.45

resources and provides more accurate market price for mineral resources. Thus, the value of mineral resources given by this model is higher than that given by the DCF model and the B-S model based on fixed volatility. This conclusion is consistent with the results shown in Tables 1 to 4, demonstrating the validity of the mineral resources pricing based on the Mix-Copula-EGARCH model.

4. Conclusion

In the early stage of resource exploitation, a large amount of capital needs to be invested, causing the investment decision-making to face enormous investment risks. In this regard, effective pricing method of mineral resources become particularly important. In this paper, according to the concept of real options, the mineral resource pricing model based on the Mix-Copula-EGARCH model is established by adopting the widely-used EGARCH model in financial research for the estimation of price fluctuations in mineral resources, combining the advantages of the Copula function for the construction of the Mix-Copula function and utilizing the risk-neutral martingale pricing method. By taking the four mineral resources of zinc, nickel, copper and tin as empirical objects, the Monte Carlo method is used to analyze them, and compare with the conventional DCF model and B-S model. The empirical results show that the conventional resource asset pricing methods

fail to accurately characterize the changes of mining prices and underestimate the value of mineral resources. The model proposed in this paper can efficiently reflect the rights and benefits of mineral resource owners if compared with the conventional methods, and better reflect the market value of mineral resources. As shown in the research results in this paper, for exhaustive resources such as mineral resources, the degree of scarcity is bound to increase gradually, thus causing further expanded price fluctuations of mineral resources. Therefore, the adoption of a time-varying volatility model in estimations is more in line with the actual situation. At the same time, unlike those conventional linear correlation methods, the Mix-Copula function adopted in this paper owns great advantages in characterizing the correlations of mineral resources assets, which can improve the accuracy of mineral resources pricing and enrich researches on the related structures of mineral resources. Mineral resources are only one of many natural resources. This paper will extend the model to value analysis of other types of natural resources, further consider the impacts of cost, interest rate and other uncertain factors on natural resource value, and build a more realistic and practical evaluation method of natural resource value.

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