# Velocity-Based Modeling and Localization of Leakage Points in Asphalt Concrete Core Walls

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**Abstract: Local leakage in asphalt concrete core walls is usually concealed, and the spatial position of the leakage outlet is difficult to determine directly. To locate hidden leakage points using downstream flow information, this study develops a velocity-based localization model for asphalt concrete core walls. A three-dimensional equivalent seepage model was established to generate virtual velocity measurements in the downstream sand–gravel transition zone. The leakage outlet was simplified as a point source, and a relationship model was constructed between measuring-point velocity, measuring-point coordinates, and leakage-point position. The localization problem was then formulated as a nonlinear least-squares problem and solved using the Levenberg–Marquardt algorithm. The results show that the proposed model can use finite downstream velocity measurements to estimate the leakage-point location under idealized numerical conditions. The calculated location agrees well with the preset leakage position, indicating that the downstream velocity data contain effective spatial information for leakage-point localization. The proposed model provides a theoretical basis for transforming virtual velocity measurements into subsequent borehole-based detection schemes for concealed leakage in asphalt concrete core wall dams.**

**Keywords: Asphalt Concrete Core Wall; Leakage-Point Localization; Velocity Measurement; Localization Model; Levenberg-Marquardt Algorithm**

## 1. Introduction

Leakage localization in asphalt concrete core wall dams remains difficult because the core wall is buried inside the dam body and leakage outlets are usually not directly observable at the downstream surface. Asphalt concrete cores are widely used as impervious elements in rockfill dams, but reported engineering cases have shown that defects in the core wall, abnormal transition-zone permeability, or insufficient drainage capacity may alter the downstream seepage field and raise dam-safety concerns [1]. Therefore, beyond detecting the occurrence of leakage, determining the possible spatial position of the leakage source is important for targeted inspection and remediation.

A wide range of methods has been developed for seepage detection in embankment dams, including piezometric monitoring, drainage discharge analysis, tracer tests, temperature monitoring, distributed fiber-optic sensing, electrical resistivity tomography, and infrared thermography [2-6]. Tracer tests can help identify leakage paths and estimate leakage discharge, while thermal and fiber-optic methods use temperature anomalies to infer seepage processes [3-5]. Electrical resistivity tomography can image internal defects or seepage-related zones from electrical-property contrasts [2,6]. These methods provide valuable information for anomaly detection, but their interpretation may remain non-unique when the dam materials are heterogeneous and the leakage path is deeply embedded.

Inverse analysis offers another route for leakage diagnosis because it attempts to infer hidden hydraulic features from observable responses. Hydraulic tomography, for example, uses a sequence of hydraulic tests and observation data to estimate hydraulic-property distributions, and has been introduced for diagnosing concentrated leakage channels embedded in dam foundations [7-9]. However, hydraulic tomography generally targets high-permeability zones or hydraulic-conductivity fields and requires suitable pumping and observation conditions. Its implementation in operating dams may be limited by the availability of boreholes, boundary uncertainty, and the cost of hydraulic testing.

Compared with indirect anomaly indicators, local flow-velocity information can provide a more direct description of seepage movement. Groundwater velocity measurement and vertical flowmeter logging have been used in hydrogeological investigations to quantify flow direction and magnitude inside boreholes and to support interpretation of subsurface hydraulic conditions [10,11]. For asphalt concrete core wall dams, such velocity information may be obtained from boreholes arranged in the downstream sand-gravel transition zone. If the relationship between measured velocity and leakage-source position can be established, borehole velocity data may provide useful constraints for leakage-point localization.

Nevertheless, existing studies have rarely formulated a quantitative relationship between finite downstream velocity measurements and the spatial coordinates of a leakage point in an asphalt concrete core wall. Most available methods focus on detecting seepage anomalies or identifying leakage pathways, whereas the use of local velocity responses as quantitative observations for leakage-point localization remains insufficiently explored. In particular, the sand-gravel transition zone provides a preferential passage for leakage water after a core-wall defect occurs, and its velocity field may contain information related to the location of the leakage source.

To address this issue, this study develops a preliminary velocity-based localization method for leakage points in asphalt concrete core walls. A three-dimensional equivalent seepage model is established to generate virtual velocity measurements in the downstream sand-gravel transition zone. The leakage outlet is simplified as a point source, and a relationship model between measuring-point velocity, measuring-point coordinates, and leakage-point position is constructed. The localization problem is then formulated as a nonlinear least-squares problem and solved using the Levenberg-Marquardt algorithm [12,13]. The purpose of this study is to verify, under idealized conditions of a homogeneous medium and a single leakage point, whether limited downstream velocity data can be used to preliminarily identify the spatial position of a core-wall leakage point and to provide a basis for future borehole-based velocity measurement schemes.

## 2. Numerical Model and Virtual Velocity Data

### 2.1 Equivalent Seepage Model

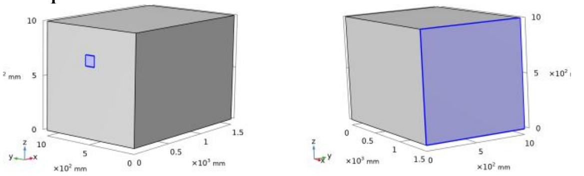
The numerical model in this section was used to generate controlled velocity observations for testing the localization method. Therefore, the purpose of the model is not to reproduce all field-scale seepage complexities, but to provide a known leakage condition and a corresponding set of downstream velocity data. This allows the identified leakage-point location to be compared with the preset leakage condition in the numerical model.

To obtain the velocity information required for localization, a three-dimensional local seepage model was established for the sand-gravel transition zone downstream of the asphalt concrete core wall. Since the focus of this study is the macroscopic velocity response rather than pore-scale flow, the sand-gravel transition zone was treated as an equivalent continuous porous medium. This assumption reduces geometric complexity and improves computational efficiency while retaining the overall seepage characteristics of the transition material [11,12,14].

The model dimensions were 1500 mm × 1000 mm × 1000 mm. The coordinate system was defined as follows: the X-axis represents the upstream-downstream direction, the Y-axis represents the dam-axis direction, and the Z-axis represents the elevation direction. The local leakage path was represented by a rectangular inflow boundary on the upstream side of the model, while the downstream side was set as a pressure outlet.

A 100 mm × 100 mm rectangular inflow boundary was assigned on the upstream side to simulate the concentrated leakage path caused by a local defect in the core wall, as shown in Figure 1(a). A normal inflow velocity of 0.05 m/s was applied at the inlet. The downstream boundary was set as a pressure outlet with a pressure of 0 Pa, as shown in Figure 1(b). The remaining boundaries were treated as no-flow or symmetry boundaries, so that seepage mainly developed from the inlet toward the outlet. Gravity was included in the calculation to account for the influence of elevation on the hydraulic head distribution. As shown in Figure 2, the model was discretized using free tetrahedral elements with physics-controlled meshing. Considering the relatively large velocity gradient near the inlet, a refined mesh level was adopted.

Two orthogonal virtual survey lines were arranged to extract velocity data for localization. The first line was aligned with the center of the leakage outlet and extended along the elevation direction, with endpoints at (500, 500, 1000) and (500, 500, 0). The second line was arranged along the dam-axis direction, with endpoints at (500, 0, 500) and (500, 1000, 500), and was located at Z = 500 mm. These two lines were used to obtain velocity information in the vertical and lateral directions, respectively. The two virtual survey lines can be regarded as an idealized representation of possible borehole-based observations. The elevation-direction line corresponds to a layered measurement profile in a single borehole, whereas the dam-axis-direction line corresponds to velocity measurements obtained from several boreholes at the same suspected elevation. In this study, however, these data are numerical velocity samples rather than field borehole measurements.



(a) Inlet of the Local Leakage Model (b) Outlet of the Local Leakage Model

**Figure 1. Local Seepage Model**

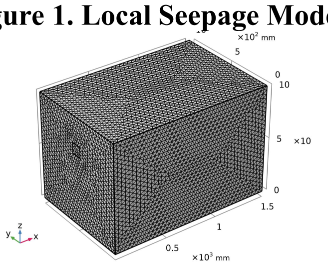


**Figure 2. Schematic Diagram of Mesh Generation**

### 2.2 Governing Assumptions and Parameter Settings

The seepage flow in the sand-gravel transition zone was regarded as low-velocity steady flow in a saturated porous medium. Darcy's law [10] was used to describe the relationship between seepage velocity and hydraulic gradient:

$$v=-K\nabla h \tag{1}$$

where $v$ is the Darcy seepage velocity vector, $K$ is the hydraulic conductivity, $h$ is the total hydraulic head, and $\nabla h$ is the hydraulic gradient. When expressed in terms of intrinsic permeability, the Darcy velocity can be written as:

$$u=-\frac{k}{\mu}\nabla p \tag{2}$$

where $u$ is the Darcy velocity, $k$ is the intrinsic permeability of the porous medium, $\mu$ is the dynamic viscosity of the fluid, and $p$ is the pore-water pressure. Water was assumed to be incompressible, and the dynamic viscosity was taken as $1.0\times10^{-3}$ Pa·s.

The intrinsic permeability of the sand-gravel material was estimated using the Kozeny-Carman model [11]:

$$k=\frac{d_p^2 n^3}{180(1-n)^2} \tag{3}$$

where $d_p$ is the representative particle size and $n$ is the porosity. In this study, $d_p$ = 10 mm and $n$ = 0.3. The sand-gravel transition zone was assumed to be homogeneous and isotropic, and only a single stable leakage inlet was considered. Heterogeneity, non-Darcy flow, and the coupling effects of multiple leakage points were not included in this basic verification model.

## 3. Velocity-Based Leakage-Point Localization Method

The basic principle of the proposed method is that a localized leakage outlet acts as a hydraulic source controlling the downstream seepage field. Under steady seepage conditions, the velocity measured at a downstream point is related to the hydraulic gradient, the seepage path length, and the permeability of the transition material. Therefore, if velocity responses are obtained at several downstream measuring points, the hidden leakage-point location can be estimated by searching for the source position that best explains the observed velocity distribution.

The local leakage outlet of the core wall was simplified as a point source $P(x_p, y_p, z_p)$. The coordinates of the downstream virtual measuring points are denoted by $(x_i, y_i, z_i)$, $i$ = 1, 2, …, $n$, and the extracted Darcy velocity at each point is denoted by $v_i$, $i$ = 1, 2, …, $n$.

The spatial distance from measuring point i to the leakage point P is expressed as:

$$L_i=\sqrt{(x_i-x_p)^2+(y_i-y_p)^2+(z_i-z_p)^2} \tag{4}$$

Under simplified conditions, the velocity at a measuring point can be regarded as the combined response of the hydraulic-head difference between the leakage point and the measuring point, the seepage path length, and the permeability of the medium. According to Darcy's law, the theoretical velocity can be written as:

$$\hat{v}_i=K\frac{\Delta h}{L_i} \quad (5)$$

where $\hat{v}_i$ is the theoretically calculated velocity at measuring point i, $K$ is the hydraulic conductivity of the sand-gravel material, $\Delta h$ is the equivalent hydraulic-head difference between the leakage point and the measuring-point region, and $L_i$ is the spatial distance from the measuring point to the leakage point.

The objective of localization is to find a set of parameters that minimizes the difference between the theoretical velocities and the velocities extracted from the virtual measuring points. The residual is defined as:

$$r_i=\hat{v}_i-v_i \quad (6)$$

The objective function can then be written as:

$$S(\theta)=\sum_{i=1}^{n} r_i^2 \quad (7)$$

where $\theta$ is the parameter vector to be identified. In general, the parameter vector can be expressed as:

$$\theta=[x_p,y_p,z_p,\Delta h]^T \quad (8)$$

Considering that the leakage inlet in the present model is located on the upstream side of the core wall, and that this study mainly focuses on the dam-axis and elevation coordinates of the leakage point, $x_p$ was constrained near the core wall. In the calculation, $x_p = 0$ was adopted, and $y_p$, $z_p$, and $\Delta h$ were identified as the main unknown parameters.

The above model is a nonlinear least-squares problem. The Levenberg-Marquardt algorithm was adopted because it combines the characteristics of the Gauss-Newton method and the steepest descent method, and can improve the stability of nonlinear parameter estimation by adaptively adjusting a damping factor [12,14].

In the Levenberg–Marquardt iteration, the Jacobian matrix represents the sensitivity of the residuals to the unknown parameters. In this problem, it describes how the calculated velocity changes when the assumed leakage-point coordinates or equivalent hydraulic-head difference are updated. Therefore, the Jacobian matrix links the spatial movement of the assumed leakage source to the variation in the velocity residuals.

At the k-th iteration, let the parameter vector be $\theta^{(k)}$, the residual vector be $r(\theta^{(k)})$, and the Jacobian matrix be $J(\theta^{(k)})$. The parameter increment $\Delta\theta^{(k)}$ can be obtained from:

$$[J^TJ+\lambda I]\Delta\theta=-J^Tr \quad (9)$$

where $\lambda$ is the damping factor and $I$ is the identity matrix. The parameter vector is updated as:

$$\theta^{(k+1)}=\theta^{(k)}+\Delta\theta^{(k)} \quad (10)$$

If the objective function decreases after the update, the update is accepted and the damping factor is reduced. If the objective function does not decrease, the update is rejected and the damping factor is increased. The iterative process continues until the parameter increment or the change in the objective function satisfies the convergence criterion.

## 4. Leakage-point Localization Results and Error Analysis

To verify the feasibility and fitting performance of the proposed leakage-source identification method based on velocity measurements along virtual survey lines, systematic localization calculations were conducted using simulation data generated from the three-dimensional seepage model. Virtual measuring points were arranged in different directions to extract key information from the seepage field, and the Levenberg–Marquardt optimization algorithm was adopted to achieve preliminary localization of the leakage point.

In the three-dimensional seepage model, measuring-point networks were systematically arranged along two orthogonal directions. The measuring points in the depth direction were distributed along the Z-axis, with 10 points covering the range from the top to the bottom of the model, to capture the variation in seepage velocity with depth. Another 10 measuring points were uniformly arranged along the Y-axis in the dam-axis direction to obtain the velocity distribution along the dam axis. The detailed data are listed in Table 1.

The velocity curves extracted from the two virtual survey lines are shown in Figure 3. The elevation-direction profile shows the vertical velocity variation, whereas the dam-axis-direction profile shows the lateral velocity variation at the same elevation.

Before localization, the two groups of velocity data were used to provide complementary spatial constraints. The elevation-direction measuring points mainly reflect the velocity variation along the Z-axis and therefore constrain the possible vertical range of the leakage point. The dam-axis-direction measuring points reflect the lateral variation of velocity at the same elevation and therefore constrain the possible Y-coordinate of the leakage point. Compared with a single survey line, the combination of two orthogonal velocity profiles reduces the non-uniqueness of the localization problem.

**Table 1. Coordinates and Seepage Velocities of Virtual Measuring Points**

| Survey-line direction | Measuring-point coordinates (mm) | Seepage velocity (mm/s) |
|---|---|---|
| Elevation direction | (500, 500, 899.75) | 0.650 |
| | (500, 500, 794.95) | 0.660 |
| | (500, 500, 699.40) | 0.659 |
| | (500, 500, 599.66) | 0.633 |
| | (500, 500, 499.53) | 0.587 |
| | (500, 500, 399.61) | 0.531 |
| | (500, 500, 298.30) | 0.476 |
| | (500, 500, 201.80) | 0.435 |
| | (500, 500, 99.53) | 0.406 |
| | (500, 500, 0) | 0.397 |
| Dam-axis direction | (500, 101.05, 500) | 0.493 |
| | (500, 201.41, 500) | 0.517 |
| | (500, 301.27, 500) | 0.550 |
| | (500, 402.76, 500) | 0.578 |
| | (500, 499.93, 500) | 0.588 |
| | (500, 600.90, 500) | 0.576 |
| | (500, 702.16, 500) | 0.549 |
| | (500, 801.08, 500) | 0.517 |
| | (500, 905.02, 500) | 0.492 |
| | (500, 1000, 500) | 0.483 |

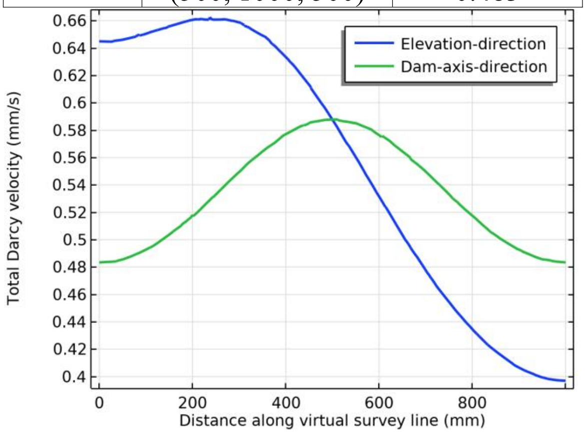



**Figure 3. Velocity Variation along the two Virtual Survey Lines**

The measured coordinates and velocity data were then introduced into the identification model. Considering that leakage sources in practical engineering are generally located near the core wall, the X-coordinate was constrained to be 0, and the Y- and Z-coordinates were identified as the main location parameters. Reasonable parameter bounds were set in the algorithm to ensure the physical plausibility of the solution. After iterative calculation, the identified leakage-source coordinate was (0, 500.3, 758.5) mm. The localization result agrees well with the preset leakage condition in the numerical model. The mean relative fitting error between the calculated velocities and the virtual velocity data was 2.2%, and the residual sum of squares was $2.38\times10^{-9}$, indicating that the proposed model can reproduce the velocity observations under the idealized numerical conditions. Here, the fitting error refers to the discrepancy between the theoretical velocities calculated by the localization model and the virtual velocities extracted from the numerical seepage model. The identified Y-coordinate is close to the central region of the dam-axis-direction measuring line, which is consistent with the position of the velocity peak in the lateral velocity profile. The identified Z-coordinate falls within the elevation range where relatively high velocities are observed along the elevation-direction profile. This consistency indicates that the optimization result is not only a numerical fit, but also agrees with the physical velocity-response characteristics obtained from the seepage model.

This case preliminarily verifies that limited virtual measuring-point velocity data can provide useful constraints for leakage-point localization under idealized numerical conditions. By combining seepage velocity and measuring-point coordinates, the spatial position of the leakage point can be preliminarily determined, providing a reference for subsequent borehole-based velocity measurement studies.

## 5. Discussion and Engineering Application

### 5.1 Influence of Measuring-point Layout

The proposed localization method depends on the spatial coverage of velocity measurements. If only elevation-direction data are used, the vertical influence range of leakage can be constrained, but the dam-axis coordinate may remain uncertain. If only dam-axis-direction data are used, the lateral projection can be indicated, but the elevation coordinate may be insufficiently constrained. Therefore, orthogonal measurement layouts are beneficial for reducing non-uniqueness in leakage-point localization.

In practical applications, the measuring range should cover the suspected leakage region and a certain range on both sides. An excessively narrow range may fail to capture the complete attenuation trend, while overly large spacing between points may miss the velocity peak. Therefore, the spacing and number of measuring points should be designed according to the expected leakage scale, the allowable drilling cost, and the required localization accuracy.

A practical measurement strategy can therefore be divided into two stages. The first stage is vertical screening, in which layered measurements in one or several control boreholes are used to determine the elevation interval with relatively high velocity. The second stage is lateral refinement, in which additional boreholes are arranged near the suspected elevation to compare velocities along the dam-axis direction. This staged strategy can reduce unnecessary boreholes and improve the efficiency of leakage-point localization.

## 5.2 Potential Transformation to Borehole Velocity Measurement

Although this study uses virtual measuring-point data, the proposed idea can be transformed into a borehole-based velocity measurement scheme. A single borehole with layered velocity measurements may provide an elevation-direction velocity profile, while multiple boreholes at the same suspected leakage elevation may provide a dam-axis-direction velocity profile. In this way, the virtual survey lines used in the numerical model can be related to feasible field measurements.

Borehole or flowmeter-based velocity measurements have been used in hydrogeological investigations to characterize local groundwater flow conditions [13]. For dam leakage detection, such measurements should be combined with piezometric monitoring, drainage discharge observations, tracer tests, temperature monitoring, and geophysical surveys. Multi-source information can help reduce the uncertainty caused by local heterogeneity and measurement errors.

## 5.3 Limitations and Prospects

This study is a preliminary numerical verification under idealized conditions. The sand-gravel transition zone was assumed to be homogeneous and isotropic, only one leakage point was considered, and the velocity data were extracted from a numerical model without measurement noise. In real dams, the gradation, compaction state, and pore structure of transition materials may vary spatially, causing the actual velocity distribution to deviate from the ideal model.

In addition, borehole drilling may change local boundary conditions and induce flow convergence or bypass flow near the borehole wall. Instrument accuracy, measuring-point positioning error, and the existence of multiple leakage points may also influence the localization result. Future studies should introduce measurement noise, different measuring-point layouts, heterogeneous permeability fields, and multiple leakage points to evaluate the robustness of the proposed method. Laboratory model tests and field data are also needed to further verify the method.

# 6. Conclusions

This study developed a velocity-based localization model for leakage points in asphalt concrete core walls using virtual velocity measurements in the downstream sand–gravel transition zone. The main conclusions are as follows.

(1) A relationship model was established between measuring-point velocity, measuring-point coordinates, and leakage-point position by simplifying the leakage outlet as a point source. The model provides a quantitative framework for using finite downstream velocity data to estimate the spatial location of a hidden leakage point.

(2) The leakage-point localization problem was formulated as a nonlinear least-squares problem and solved using the Levenberg–Marquardt algorithm. By constraining the upstream–downstream coordinate near the core-wall plane, the dam-axis coordinate, elevation coordinate, and equivalent hydraulic-head difference were identified as the main unknown parameters.

(3) The two orthogonal velocity profiles provide complementary constraints for model solution. The elevation-direction velocity data mainly constrain the vertical coordinate of the leakage point, whereas the dam-axis-direction velocity data mainly constrain its lateral coordinate, reducing the non-uniqueness of the localization problem.

(4) Based on finite virtual velocity measurements, the proposed model identified a leakage-point location that agreed well with the preset numerical condition. The mean relative fitting error and residual sum of squares indicate

that the model can reproduce the virtual velocity observations under idealized conditions.

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